Estimating fault masking using Squeeziness based on Rényi's entropy

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ABSTRACT

Squeeziness is an Information Theory notion that has been proven to strongly correlate with the likelihood of Failed Error Propagation (FEP). This allows us to estimate the FEP of a certain system by computing its Squeeziness. The original notion of Squeeziness is based on the classical notion of entropy defined by Shannon. In this paper we study alternative notions of Squeeziness based on a more general notion of entropy introduced by Rényi. In contrast to Shannon's entropy, which is univocally defined, Rényi's entropy depends on a parameter α. We define Squeeziness by using Rényi's entropy and analyse the correlation of the different notions of Squeeziness with the likelihood of FEP. Our experiments showed that although $\alpha = 1$, corresponding to Shannon's entropy, induces good correlations, there are values of α showing better correlations.

CCS CONCEPTS

• **Software and its engineering** → **Software testing and debugging**; • **Mathematics of computing** → *Information theory*; • **Theory of computation** → Abstract machines;

KEYWORDS

Formal approaches to testing; Information Theory; Failed Error Propagation; Rényi's entropy

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1 INTRODUCTION

Software Testing [2, 26] is the main validation technique to detect faults in software systems. Traditionally, Software Testing was a repertory of *informal* techniques. However, it has been shown that it is possible to *formalise* it [9, 16] and there are many tools supporting the theoretical frameworks [23, 29]. One of the main scenarios where formal methods for testing are fundamental is black-box testing. In this scenario, the tester observes the reaction of the System Under Test (SUT) to the provided inputs without having access to the internal structure of the SUT. Many formal approaches have been developed for black-box testing but there are some shortcomings difficult to overcome. Among them, we can stand out Failed Error Propagation (FEP).

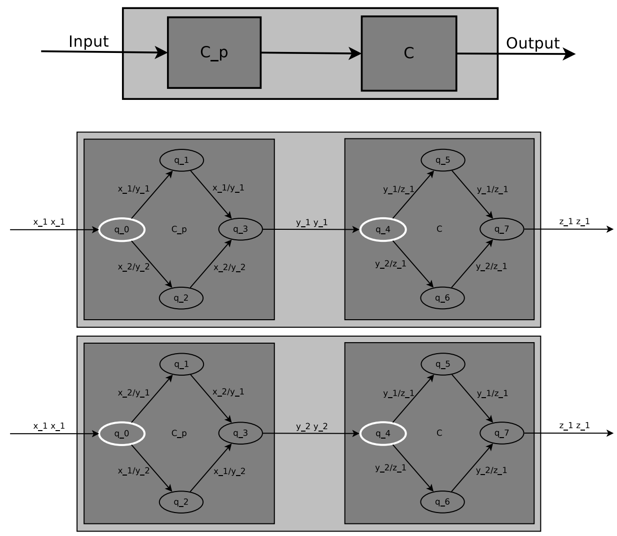

Figure 1: Representation of our testing scenario.

In terms of the RIP model [2] (stating that three conditions must be present for a failure to be observed: Reachability, Infection and Propagation), we might *reach* a fault such that the *infection* is not *propagated* to the final (observable) state. The lack of access to the internal structure of the SUT negates the tester the possibility to detect these faults in a black-box scenario.[1]

It might be thought that if the previous faults do not alter the outputs, then we should not worry about them. However, more complex forms of FEP consist in faults whose errors do not propagate to the outputs in some cases, but they generate wrong outputs in other cases. These forms of FEP are specially dangerous because their detection depends on executing the right test, the one that propagates the error to the output. However, if the right test is not in the selected test suite (maybe because it will only detect this fault and the other tests could detect more than one fault at the same time), then the error will remain undetected. Finally, one last dangerous form of FEP appears in systems where the first execution of the faulty code does not generate the wrong output, but it still corrupts the internal state of the system. This can lead to a wrong output after some time.

[1]These faults can be detected in a white-box scenario because the code is available and then we can *follow* the produced error.

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An example of FEP is illustrated in Figure 1 (this is the scenario that we consider in this paper). We have a component C receiving a sequence of inputs from another component C_P. C and C_P can be modelled as Finite State Machines (FSMs). We assume that these values (sent by C_P to C) are not directly observed by the tester and that C produces a sequence of outputs that are either observed during testing or are received by another component. In this context, C_P could produce an unexpected sequence but component C could map the expected and unexpected sequences to the same output sequence: C would introduce a form of FEP that makes it more difficult to find faults in C_P. These faults could be unleashed if we compose C_P with a different component C'. Assume that we want to implement the component C_p given in the middle part of Figure 1 and that this component will be paired with component C. In this setting, it will be difficult to unmask a faulty implementation of C_p, such as the one shown in the lower part of Figure 1, because C returns the same response, the sequence z_1z_1, to the sequences y_1y_1 (produced by a correct implementation of C_p receiving x_1x_1) and y_2y_2 (produced by a faulty implementation of C_p also receiving x_1x_1). Note, as we already said, that a tester will not be able to observe whether the sequence provided to C is y_1y_1 or y_2y_2.

Since FEP cannot be directly detected, it is important to estimate its likelihood. Squeeziness [3, 11] was used to estimate FEP in a white-box scenario because there is a strong rank correlation between Squeeziness and FEP. Specifically, Squeeziness is a measure of the loss of information (entropy) that happens in a channel (in this case, the SUT) that takes inputs and return outputs. The idea behind it is that if the SUT maps two or more inputs to the same output then this channel (the SUT) can lead to a loss of information: if we know the program output then we may not know the program input that caused this. In recent work, and this is the notion that we consider in this paper, Squeeziness has been adapted to a black-box scenario [21], where the specification of the SUT is given as a FSM. The observable behaviour of an FSM is given by the set of input/output sequences, usually called *traces*, that label paths from the initial state of the system.

Squeeziness was always defined using Shannon's entropy [30], although there are many alternative notions to define what is intended as entropy. Rényi's entropy [28] provides an infinite family of *entropies* because its definition is parameterised by a positive real value α. A good property of this general notion is that Shannon's entropy, as well as other notions appearing in the literature, are specific cases of this generalisation (Shannon's entropy corresponds to $\alpha = 1$). The main goal of this paper is to generalise Squeeziness to deal with Rényi's entropy and explore how it improves the performance of Squeeziness. We compute the correlations between Squeeziness based on Rényi's entropy and the likelihood of FEP, for FSMs with different number of states. First, we consider the values for the extreme cases ($\alpha \in \{0, 1, \infty\}$). Afterwards, we use uniformly distributed values in the ranges $[0, 1]$, $[1, 10]$ and $[10, 100]$. The obtained results were very promising. There is a correlation between FEP and Rényi's Squeeziness that ranges between 0.5 and 0.9. Furthermore, the best correlations are obtained with $\alpha \in (2, 3)$, where the correlations range between 0.75 and 0.9.

The rest of the paper is organised as follows. In Section 2 we explain some basic concepts on FSMs and present the main definitions of previous work. In Section 3 we develop the adaptation of the previously explained theory to the new concept of entropy. In Section 4 we explain the experiments performed to evaluate our newly developed theory. In Section 5 we present our conclusions and some lines of future work.

2 PRELIMINARIES

In this section we will present some concepts that are required to understand the work presented in this paper. These concepts are standard in classical work on testing from FSMs [22]. Most of them are based on the original sources, while some notation is adapted to facilitate the formulation of subsequent definitions.

2.1 Basic concepts

Given a set A, we let A^* denote the set of finite sequences of elements of A. $\epsilon \in A^*$ denotes the empty sequence. We let A^+ denote the set of non-empty sequences of elements of A. A^k denotes the set of sequences with length $k \geq 1$. We let $|A|$ denote the cardinal of set A. Given a sequence $\sigma \in A^*$, we have that $|\sigma|$ denotes its length. Given a sequence $\sigma \in A^*$ and $a \in A$, we have that σa denotes the sequence σ followed by a and $a\sigma$ denotes the sequence σ preceded by a.

Throughout this paper we let I be the set of input actions and O be the set of output actions. It is important to differentiate between input actions and *inputs* of the system. In our context, an input of a system will be a non-empty sequence of input actions, that is, an element of I^+ (similarly for outputs and output actions).

A *Finite State Machine* is a (finite) labelled transition system in which transitions are labelled by an input/output pair. We use this formalism to define processes.

Definition 2.1. We say that $M = (Q, q_{in}, I, O, T)$ is a *Finite State Machine* (FSM), where Q is a finite set of states, $q_{in} \in Q$ is the initial state, I is a finite set of input actions, O is a finite set of output actions, and $T \subseteq Q \times (I \times O) \times Q$ is the transition relation. A transition $(q, (i, o), q') \in T$, also denoted by $q \xrightarrow{i/o} q'$ or by $(q, i/o, q')$, means that from state q after receiving input i it is possible to move to state q' and produce output o.

We say that M is *deterministic* if for all $q \in Q$ and $i \in I$ there exists at most one pair $(q', o) \in Q \times O$ such that $(q, i/o, q') \in T$. In this paper we consider deterministic FSMs.

An FSM can be represented by a diagram in which nodes represent states of the FSM and transitions are represented by arcs between the nodes. We use a double circle to denote the initial state.

As stated in the previous definition, we consider that FSMs are deterministic. This restriction is taken to mimic the white-box scenario where Squeeziness was originally introduced and considered, as usual, that programs are deterministic.

Definition 2.2. Let $M = (Q, q_{in}, I, O, T)$ be an FSM. We say that $(i_1, o_1) \dots (i_k, o_k) \in (I \times O)^*$ is a *trace* of M if there exist states $q_1 \dots q_k \in Q$ such that for all $1 \leq j \leq k$ we have $(q_{j-1}, i_j/o_j, q_j) \in T$, where $q_0 = q_{in}$. Let $s = i_1 \dots i_k \in I^*$ be a sequence of input actions. We define $\mathtt{out}_M(s)$ as the set

$$\{o_1 \dots o_k \in O^* | (i_1/o_1) \dots (i_k, o_k) \text{ trace of } M\}$$

Note that if M is deterministic, then this set is either empty or a singleton. In the last case we will sometimes write $\texttt{out}_M(s) = o_1, \ldots, o_k$.

We define $\texttt{dom}_M$ as the set $\{s \in I^* | \texttt{out}_M(s) \neq \emptyset\}$. Similarly, we define $\texttt{image}_M$ as the set

$$\{o_1 \ldots o_k \in O^* | \exists s \in I^* : o_1 \ldots o_k \in \texttt{out}_M(s)\}$$

We denote by $\texttt{dom}_{M,k}$ the set $\texttt{dom}_M \cap I^k$. Similarly, we denote by $\texttt{image}_{M,k}$ the set $\texttt{image}_M \cap O^k$.

2.2 Shannon-based Squeeziness in a black-box setting

Squeeziness has been used to estimate the existence of FEP in a black-box scenario [21]. In order to do that, FSMs represent specifications as functions that transform sequences of input actions into sequences of output actions. Those inputs will belong to $\texttt{dom}_M \subseteq I^*$, while outputs will belong to $\texttt{image}_M \subseteq O^*$. *Projections* of these functions restrict the function to sequences of input actions of length $k > 0$. Finally, we review the notion of *collision*, which happens when two different inputs produce the same output.

Definition 2.3. Let $M = (Q, q_{in}, I, O, T)$ be an FSM. We define $f_M : \texttt{dom}_M \longrightarrow \texttt{image}_M$ as the function such that for all $s \in \texttt{dom}_M$ we have $f_M(s) = \texttt{out}_M(s)$.

Let $k > 0$. We define $f_{M,k}$ to be the function $f_M \cap (I^k \times O^k)$, where we use the function f_M to denote the associated set of pairs. Let $t \in \texttt{image}_M$. We define $f_M^{-1}(t)$ to be the set $\{s \in I^* | f_M(s) = t\}$.

Let $s_1, s_2 \in I^*$. We say that s_1 and s_2 collide for M if $s_1 \neq s_2$ and $f_M(s_1) = f_M(s_2)$.

Squeeziness represents the amount of information lost by a function. Thus, Squeeziness for an FSM was defined as the Squeeziness of the function that represents this FSM. In order to properly compute it, it was necessary to define how inputs are chosen and outputs are returned. A probabilistic view, where a random variable is associated with each set of relevant inputs/outputs, was considered. Specifically, a random variable was associated with the set of inputs/outputs of a certain length (that is, there are different random variables associated with $I^1, I^2, \ldots; O^1, O^2, \ldots$). Since $\texttt{dom}_{M,k}$ includes the inputs of length equal to k that M can perform and $\texttt{image}_{M,k}$ includes the outputs of length equal to k that M can produce after receiving an element of $\texttt{dom}_{M,k}$, random variables ranging over each set are defined as $\xi_{\texttt{dom}_{M,k}}$ and $\xi_{\texttt{image}_{M,k}}$, respectively. With these random variables, the concept of Squeeziness for FSMs was defined.

Definition 2.4. Let S be a set and ξ_S be a random variable over S. We denote by σ_{ξ_S} the probability distribution induced by ξ_S.

Let $M = (Q, q_{in}, I, O, T)$ be an FSM and $k > 0$. Let us consider two random variables $\xi_{\texttt{dom}_{M,k}}$ and $\xi_{\texttt{image}_{M,k}}$ ranging, respectively, over the domain and image of $f_{M,k}$. The Squeeziness of M at length k is defined as

$$\texttt{Sq}_k(M) = \mathcal{H}(\xi_{\texttt{dom}_{M,k}}) - \mathcal{H}(\xi_{\texttt{image}_{M,k}})$$

where $\mathcal{H}(\xi_S)$ denotes the (Shannon's) entropy of the random variable ξ_S that ranges over the set S, which is defined as

$$\mathcal{H}(\xi_S) = -\sum_{s \in S} \sigma_{\xi_S}(s) \cdot \log_2(\sigma_{\xi_S}(s))$$

There is an important remark concerning random variables associated with inputs and outputs: given an FSM M, $k > 0$ and a random variable $\xi_{\texttt{dom}_{M,k}}$, we have that the probability distribution of the random variable $\xi_{\texttt{image}_{M,k}}$ is completely determined. This is because for each element $t \in \texttt{image}_{M,k}$ we have that

$$\sigma_{\xi_{\texttt{image}_{M,k}}}(t) = \sum_{s \in f_M^{-1}(t)} \sigma_{\xi_{\texttt{dom}_{M,k}}}(s)$$

Therefore, the formulation of Squeeziness is

$$\texttt{Sq}_k(M) = -\sum_{t \in \texttt{image}_{M,k}} \left(\sum_{s \in f_M^{-1}(t)} \sigma_{\xi_{\texttt{dom}_{M,k}}}(s) \right) \cdot \mathcal{R}_M(t)$$

where the term $\mathcal{R}_M(t)$ is equal to

$$\sum_{s \in f_M^{-1}(t)} \frac{\sigma_{\xi_{\texttt{dom}_{M,k}}}(s)}{\sigma_{\xi_{\texttt{dom}_{M,k}}}(f_M^{-1}(t))} \cdot \log_2 \left(\frac{\sigma_{\xi_{\texttt{dom}_{M,k}}}(s)}{\sigma_{\xi_{\texttt{dom}_{M,k}}}(f_M^{-1}(t))} \right)$$

Finally, the last concept that we will recall from previous work is *probability of collisions* (PColl [11]). In our context, fault masking (FEP) happens when the expected and faulty input sequences, received from another component, produce the same sequence t of output actions. If given an FSM M and $k > 0$ we have that there exists $t \in \texttt{image}_{M,k}$ such that $s, s' \in f_{M,k}^{-1}(t)$, with $s \neq s'$, then there is a collision. Note that collisions are a precondition of FEP.

Definition 2.5. Let M be an FSM and $k > 0$. Let $\texttt{image}_{M,k} = \{t_1, ..., t_n\}$ and for all $1 \leq i \leq n$ let $I_i = f_{M,k}^{-1}(t_i)$ and $m_i = |f_{M,k}^{-1}(t_i)|$. We have that $d = \sum_{i=1}^{n} m_i$ is the size of the input space.

Given a uniform distribution over the inputs, the probability of s and s' both being in the set I_i is equal to $p_i = \frac{m_i \cdot (m_i - 1)}{d \cdot (d-1)}$. We have that the probability of having a collision in M for sequences of length k, denoted by $\texttt{PColl}_k(M)$, is given by

$$\texttt{PColl}_k(M) = \sum_{i=1}^{n} \frac{m_i \cdot (m_i - 1)}{d \cdot (d - 1)}$$

With regard to this definition, a topic that has been already addressed is the potential to use $\texttt{PColl}_k(M)$ instead of Squeeziness. The problem with using $\texttt{PColl}_k(M)$ is that it is hard to compute. While this also applies to Squeeziness, the latter has the advantage of being an information theoretic measure. As a result, we can use Information Theory to either estimate or bound measures [6, 10], what will suffice for our task.

3 RÉNYI'S ENTROPY AND SQUEEZINESS

Previous work on Squeeziness used Shannon's entropy, but there exist alternative definitions of entropy that are worth exploring. In fact, there exists a general definition of entropy, dependent on a parameter α, called *Rényi's entropy* [28].

Definition 3.1. Let S be a set and ξ_S be a random variable over S. Let $\alpha \in \mathbb{R}_+ \backslash \{1\}$. The *Rényi's entropy* of the random variable ξ_S with respect to α, denoted by $\mathcal{H}_\alpha(\xi_S)$, is defined as:

$$\mathcal{H}_\alpha(\xi_S) = \frac{1}{1 - \alpha} \cdot \log_2 \left(\sum_{s \in S} \sigma_{\xi_S}(s)^\alpha \right)$$

Let S and T be sets and $f : S \longrightarrow T$ be a total function. Let us consider two random variables ξ_S and ξ_T ranging, respectively, over S and T, and $\alpha \in \mathbb{R}_+\backslash\{1\}$. The *Rényi's Squeeziness* of f with respect to α, denoted by $\mathsf{Sq}_\alpha(f)$, is defined as the loss of information after applying f to S taking into account α, that is, $\mathcal{H}_\alpha(\xi_S) - \mathcal{H}_\alpha(\xi_T)$.

It is well-known that when α tends to 1, Rényi's entropy becomes Shannon's entropy, that is,

$$\lim_{\alpha\rightarrow 1} \mathcal{H}_\alpha(\xi_S) = \mathcal{H}(\xi_S) = -\sum_{s\in S} \sigma_{\xi_S}(s) \cdot \log_2(\sigma_{\xi_S}(s))$$

Next, we can define the Squeeziness of an FSM using Rényi's entropy in the same way as it was defined in Definition 2.4.

Definition 3.2. Let $M = (Q, q_{in}, I, O, T)$ be an FSM and $k > 0$. Let us consider two random variables $\xi_{\mathsf{dom}_{M,k}}$ and $\xi_{\mathtt{image}_{M,k}}$ ranging, respectively, over the domain and image of $f_{M,k}$. Let $\alpha \in \mathbb{R}_+\backslash\{1\}$. Rényi's Squeeziness of M at length k with respect to α is defined as

$$\mathsf{Sq}_{\alpha,k}(M) = \mathcal{H}_\alpha(\xi_{\mathsf{dom}_{M,k}}) - \mathcal{H}_\alpha(\xi_{\mathtt{image}_{M,k}})$$

We can provide an alternative definition of Rényi's Squeeziness taking into account, as previously explained, that we have

$$\sigma_{\xi_{\mathtt{image}_{M,k}}}(t) = \sum_{s\in f_M^{-1}(t)} \sigma_{\xi_{\mathsf{dom}_{M,k}}}(s)$$

Therefore, we only need to use the probability distribution on inputs given by $\xi_{\mathsf{dom}_{M,k}}$. The proof of the following result is straightforward.

Lemma 3.3. *Let $M = (Q, q_{in}, I, O, T)$ be an FSM, $k > 0$ and $\alpha \in R_+\backslash\{1\}$. Let us consider a random variable $\xi_{\mathsf{dom}_{M,k}}$ ranging over the domain of $f_{M,k}$. We have that*

$$\mathsf{Sq}_{\alpha,k}(M) = \frac{1}{1-\alpha} \cdot \log_2\left(\frac{\sum_{s\in\mathsf{dom}_{M,k}} \left(\sigma_{\xi_{\mathsf{dom}_{M,k}}}(s)\right)^\alpha}{\sum_{t\in\mathtt{image}_{M,k}} \left(\sum_{s\in f_M^{-1}(t)} \sigma_{\xi_{\mathsf{dom}_{M,k}}}(s)\right)^\alpha}\right)$$

If α tends to 1 then we obtain Shannon's entropy [28] and we have

$$\mathsf{Sq}_{1,k}(M) = -\sum_{t\in\mathtt{image}_{M,k}} \left(\sum_{s\in f_M^{-1}(t)} \sigma_{\xi_{\mathsf{dom}_{M,k}}}(s)\right) \cdot \mathcal{R}_M(t)$$

where the term $\mathcal{R}_M(t)$ is equal to

$$\sum_{s\in f_M^{-1}(t)} \frac{\sigma_{\xi_{\mathsf{dom}_{M,k}}}(s)}{\sigma_{\xi_{\mathsf{dom}_{M,k}}}(f_M^{-1}(t))} \cdot \log_2\left(\frac{\sigma_{\xi_{\mathsf{dom}_{M,k}}}(s)}{\sigma_{\xi_{\mathsf{dom}_{M,k}}}(f_M^{-1}(t))}\right)$$

If α tends to ∞ then we obtain min-entropy [28] (that is, $\mathcal{H}_\infty(X) = -\log_2(\max_i p_i)$) and we have

$$\mathsf{Sq}_{\infty,k}(M) = \log_2\left(\frac{\max_{t\in\mathtt{image}_{M,k}} \sum_{s\in f_M^{-1}(t)} \sigma_{\xi_{\mathsf{dom}_{M,k}}}(s)}{\max_{s\in\mathsf{dom}_{M,k}} \sigma_{\xi_{\mathsf{dom}_{M,k}}}(s)}\right)$$

The proof of the previous result when α tends to 1 uses the formulation of Squeeziness given in previous work [21].

The definition of Rényi's Squeeziness keeps some of the interesting properties of the notion of Squeeziness based on Shannon's entropy [21]. The first result corresponds to the relation of the bijectivy of a function and the nullity of its Squeeziness.

Lemma 3.4. *Let $M = (Q, q_{in}, I, O, T)$ be an FSM and $k > 0$. If $f_{M,k}$ is bijective then $\mathsf{Sq}_{\alpha,k}(M) = 0$.*

The second result corresponds to the non-monotoniticy of the relationship between Squeeziness and PColl.

Lemma 3.5. *There exist FSMs M_1 and M_2 and $k > 0$ such that, for all $\alpha \in R_+\backslash\{1\}$, $\mathsf{Sq}_{\alpha,k}(M_1) \leq \mathsf{Sq}_{\alpha,k}(M_2)$ but $\mathtt{PColl}_k(M_1) > \mathtt{PColl}_k(M_2)$. In fact, the result also holds when α tends to 1 and when it tends to ∞.*

The previously defined notion of Squeeziness is parameterised by the distribution over the inputs of the function (that is, over the input sequences that the FSM can perform). If we know the actual distribution, then we can use this. If we do not know the distribution, then there is a need to choose one and we now discuss two approaches to do this.

3.1 Maximum entropy principle

We can select the distribution that maximises the entropy. If there are no further restrictions, maximum entropy is obtained with a uniform distribution [1, 12]. Then, under this distribution, the weight of a single element of $\mathsf{dom}_{M,k}$ is $\frac{1}{|\mathsf{dom}_{M,k}|}$ and the weight of the inverse image of an output $t \in \mathtt{image}_{M,k}$ is equal to $\frac{|f_M^{-1}(t)|}{|\mathsf{dom}_{M,k}|}$.

Under these assumptions, and after some algebraic manipulations, the formula for Rényi's Squeeziness becomes:

$$\mathsf{Sq}_{\alpha,k}(M) = \frac{1}{1-\alpha} \cdot \log_2\left(\frac{|\mathsf{dom}_{M,k}|}{\sum_{t\in\mathtt{image}_{M,k}} \left(|f_M^{-1}(t)|\right)^\alpha}\right)$$

As usual, we have two special cases: α tending to 1 or to ∞. If α tends to 1, then we are using Shannon's entropy and we have the following simplified formulation [21]:

$$\mathsf{Sq}_{1,k}(M) = \frac{1}{|\mathsf{dom}_{M,k}|} \cdot \sum_{t\in\mathtt{image}_{M,k}} |f_M^{-1}(t)| \cdot \log_2(|f_M^{-1}(t)|)$$

If α tends to ∞, then we are using min-entropy and, after some algebraic manipulations, we obtain the following formulation:

$$\mathsf{Sq}_{\infty,k}(M) = \log_2\left(\max_{t\in\mathtt{image}_{M,k}} |f_M^{-1}(t)|\right)$$

3.2 Maximum loss of information

Another option is to consider the worst case scenario, that is, the scenario where the probability distribution induces the maximum loss of information. In order to maximize the loss of information, we need to maximize Squeeziness. Then, the probability distribution will be the one that is uniformly distributed in the largest inverse image of an element of the outputs and zero elsewhere [11]. Formally, consider $t' \in \texttt{image}_{M,k}$ such that for all $t \in \texttt{image}_{M,k}$ we have that $|f_M^{-1}(t')| \geq |f_M^{-1}(t)|$. Then,

$$\sigma_{\xi_{\texttt{dom}_{M,k}}}(s) = \begin{cases} \frac{1}{|f_M^{-1}(t')|} & \text{if } s \in f_M^{-1}(t') \\ 0 & \text{otherwise} \end{cases}$$

Using this probability distribution, the formulation of Rényi's Squeeziness can be transformed into the following one:

$$\mathsf{Sq}_{\alpha,k}(M) = \log_2\left(|f_M^{-1}(t')|\right)$$

In this case, unlike the previous ones, Squeeziness does not depend on the value of α. In particular, the two special cases (α tending to 1 and α tending to ∞) have the same formulation.

4 EMPIRICAL EVALUATION

In order to explore the convenience of Rényi's Squeeziness, we will use the same reference measure that has been used in previous work [11, 21]: the probability of collisions (PColl as introduced in Definition 2.5). Then, our experiments will essentially compute the correlation between Rényi's Squeeziness, for different values of α, and the corresponding values of PColl.

With this methodology in mind, we asked ourselves the following research questions.

4.1 Research Questions

In order to decide whether a notion of Squeeziness based on Rényi's entropy has some scientific interest, our first research question considers whether we obtain an improvement with respect to the framework where Shannon's entropy is used.

Research Question 1. *Does there exist $\alpha \in \mathbb{R}_+ \backslash \{1\}$ whose corresponding Squeeziness correlates better with* FEP *than $\alpha = 1$? Is it unique?*

Then, in order to evaluate how the size of the FSM affects the capability of Squeeziness to detect FEP, we propose the following research question.

Research Question 2. *Is there an improvement in the capability of Squeeziness to detect cases of* FEP *when the size of the* FSM *increases?*

4.2 Experiments

In order to answer the research questions, we performed several experiments. We used 3 different sets of experimental subjects:

- Set1: 500 randomly generated FSMs with 50 states, 5 outgoing transitions from each state, and input and output alphabets of size 5.
- Set2: 3500 randomly generated FSMs with 5 outgoing transitions from each state, and input and output alphabets of size 5. This set is divided in 7 subsets, each one with 500 FSMs with the same number of states: 10, 20, 30, 40, 50, 60 and 70 states respectively.
- Set3: 241 deterministic FSMs coming from a recently collected benchmark [27], which represent real systems.

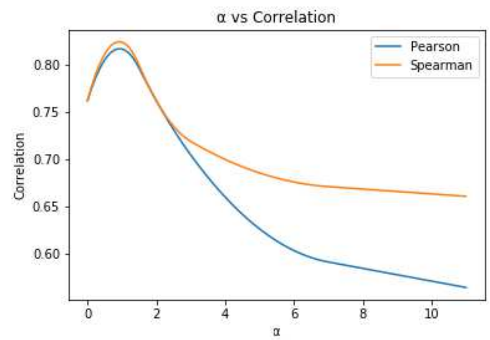

Figure 2: Initial hypothesis of the Pearson and Spearman correlations.

The code developed to perform these experiments can be found at the repositories https://github.com/Colosu/RenyiSqueeziness and https://github.com/Colosu/RenyiSqueezinessReal.

Our initial hypothesis was that $\alpha = 1$ could be the best possible value to use in the computation of Squeeziness based on Renyi's entropy. If we were able to show evidence of this hypothesis, then we could discard Renyi's entropy and stick to the original work on Squeeziness where Shannon's entropy was used. Therefore, we did a preliminary experiment where we computed the values of Squeeziness for Set1 and the extreme cases: $\alpha \in \{0, 1, \infty\}$. The best correlation between $\mathsf{Sq}_{\alpha,10}(M)$ and $\mathtt{PColl}_{10}(M)$ was obtained when $\alpha = 1$. Thus, we hypothesised that the curves showing correlation values versus α will be like the ones given in Figure 2.

In order to explore if our initial hypothesis was correct, we had to explore how the correlations perform for more values of $\alpha \in \mathbb{R}_+ \backslash \{1\}$ (specifically, we considered values of α uniformly distributed in the ranges [0, 1], [1, 10] and [10, 100]). In addition, we varied the number of states of the FSMs so that the results did not depend on a specific structure of the considered systems. In order to do that we set the following experiment. We decided to explore the correlations for FSMs with 10, 20, 30, 40, 50, 60 and 70 states (that is, Set2). Then, for each number of states we used 500 FSMs with the selected number of states, 5 outgoing transitions from each state, and input and output alphabets of size 5. Those parameters where selected in the same way as the ones used in previous work [21] so that we could properly compare the results.[2]

Then, we took each set of 500 FSMs with the same number of states and computed, for each $\alpha \in \{0, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 20, 30, 40, 50, 60, 70, 80, 90, 100, \infty\}$, the values of $\mathsf{Sq}_{\alpha,10}(M)$ and $\mathtt{PColl}_{10}(M)$, that is, we considered sequences of 10 inputs for each FSM M. Afterwards, we computed the Pearson and Spearman correlations between these values. The full results are displayed in Tables 1 and 2. Interestingly, but somehow expected, although the Pearson and Spearman correlations are different in all the cases, the difference between both correlations strongly decreases when the size of the FSMs increases.

In order to analyse all these values, we performed, for each set of FSMs with the same number of states, the cubic interpolation of

[2] We performed some experiments with different sizes of inputs and outputs alphabets and the results were essentially the same.

FSM size	10	20	30	40	50	60	70
$\alpha = \mathbf{0}$	0.486431	0.648953	0.701338	0.769901	0.762446	0.796774	0.795541
$\alpha = \mathbf{0.1}$	0.494168	0.657132	0.707142	0.776037	0.768796	0.802761	0.801591
$\alpha = \mathbf{0.2}$	0.502264	0.665466	0.712918	0.782091	0.775042	0.808578	0.807555
$\alpha = \mathbf{0.3}$	0.510699	0.673913	0.718639	0.788036	0.781146	0.814196	0.813402
$\alpha = \mathbf{0.4}$	0.51945	0.682431	0.724282	0.793846	0.787073	0.819592	0.819105
$\alpha = \mathbf{0.5}$	0.528487	0.690979	0.729829	0.7995	0.792791	0.824745	0.82464
$\alpha = \mathbf{0.6}$	0.537774	0.699512	0.735264	0.804978	0.798272	0.829641	0.829985
$\alpha = \mathbf{0.7}$	0.547273	0.70799	0.740575	0.810265	0.803492	0.834269	0.835126
$\alpha = \mathbf{0.8}$	0.556945	0.71637	0.745755	0.815351	0.808431	0.838623	0.840049
$\alpha = \mathbf{0.9}$	0.566748	0.724615	0.750799	0.820227	0.813072	0.8427	0.844744
$\alpha \to \mathbf{1}$	0.576644	0.732685	0.755706	0.824886	0.817402	0.8465	0.849204
$\alpha = \mathbf{2}$	0.666551	0.796947	0.79693	0.858781	0.842322	0.86988	0.879998
$\alpha = \mathbf{3}$	0.6879	0.814853	0.80419	0.856621	0.827749	0.85593	0.867284
$\alpha = \mathbf{4}$	0.668366	0.796306	0.769726	0.816314	0.768392	0.8129	0.81342
$\alpha = \mathbf{5}$	0.646528	0.771592	0.734235	0.778553	0.702129	0.768647	0.768431
$\alpha = \mathbf{6}$	0.629883	0.751551	0.706872	0.751479	0.652932	0.729861	0.734089
$\alpha = \mathbf{7}$	0.617827	0.736731	0.686756	0.732532	0.619978	0.699701	0.707358
$\alpha = \mathbf{8}$	0.609014	0.725885	0.671986	0.719035	0.59767	0.677549	0.686398
$\alpha = \mathbf{9}$	0.602426	0.717853	0.660975	0.709193	0.581985	0.661496	0.66999
$\alpha = \mathbf{10}$	0.597385	0.711794	0.652614	0.701865	0.570544	0.649774	0.657148
$\alpha = \mathbf{20}$	0.578372	0.690007	0.62309	0.677375	0.532961	0.612142	0.61089
$\alpha = \mathbf{30}$	0.573687	0.68491	0.617039	0.672633	0.525918	0.605177	0.601954
$\alpha = \mathbf{40}$	0.571616	0.682788	0.614691	0.670875	0.523325	0.602808	0.598745
$\alpha = \mathbf{50}$	0.570441	0.681663	0.613478	0.670023	0.522039	0.601727	0.597174
$\alpha = \mathbf{60}$	0.569685	0.680978	0.612748	0.669539	0.521291	0.601131	0.596266
$\alpha = \mathbf{70}$	0.569159	0.680523	0.612263	0.669233	0.520809	0.600758	0.595689
$\alpha = \mathbf{80}$	$-nan$	0.6802	0.611919	0.669024	0.520477	0.600504	0.595297
$\alpha = \mathbf{90}$	$-nan$	0.679959	0.611664	0.668872	0.520235	0.600319	0.595018
$\alpha = \mathbf{100}$	$-nan$	$-nan$	$-nan$	$-nan$	0.520052	0.600178	$-nan$
$\alpha \to \infty$	0.566197	0.678277	0.609975	0.667912	0.518761	0.599138	0.593596

Table 1: Pearson correlations between Rényi's Squeeziness and PColl.

FSM size	10	20	30	40	50	60	70
$\alpha = \mathbf{0}$	0.665932	0.726231	0.755532	0.750096	0.762307	0.777202	0.788155
$\alpha = \mathbf{0.1}$	0.671902	0.732722	0.762912	0.756609	0.769409	0.782764	0.793699
$\alpha = \mathbf{0.2}$	0.678459	0.739066	0.769426	0.762733	0.776362	0.789153	0.798919
$\alpha = \mathbf{0.3}$	0.684373	0.744736	0.776493	0.769181	0.783407	0.794884	0.804108
$\alpha = \mathbf{0.4}$	0.691055	0.751285	0.782856	0.775335	0.790109	0.800489	0.809323
$\alpha = \mathbf{0.5}$	0.696784	0.756789	0.789381	0.781042	0.797739	0.805876	0.813798
$\alpha = \mathbf{0.6}$	0.702745	0.762747	0.79492	0.786539	0.804071	0.811034	0.818569
$\alpha = \mathbf{0.7}$	0.708673	0.769094	0.799407	0.792022	0.8099	0.816104	0.822752
$\alpha = \mathbf{0.8}$	0.714102	0.774626	0.804144	0.797834	0.81562	0.820327	0.826531
$\alpha = \mathbf{0.9}$	0.720631	0.780229	0.809282	0.80226	0.820262	0.824522	0.830042
$\alpha \to \mathbf{1}$	0.726016	0.785447	0.812757	0.806145	0.824706	0.828592	0.833766
$\alpha = \mathbf{2}$	0.764498	0.823818	0.842291	0.837653	0.853455	0.851616	0.857426
$\alpha = \mathbf{3}$	0.764927	0.830047	0.837677	0.844951	0.846256	0.850812	0.852832
$\alpha = \mathbf{4}$	0.746544	0.818795	0.813882	0.827714	0.816204	0.833841	0.829668
$\alpha = \mathbf{5}$	0.730784	0.799161	0.783527	0.801646	0.776347	0.802588	0.79919
$\alpha = \mathbf{6}$	0.717461	0.779128	0.753262	0.777588	0.742974	0.771497	0.765878
$\alpha = \mathbf{7}$	0.707339	0.762354	0.728659	0.754509	0.715904	0.744383	0.734198
$\alpha = \mathbf{8}$	0.699611	0.748681	0.710361	0.737043	0.693262	0.724578	0.706547
$\alpha = \mathbf{9}$	0.693184	0.737617	0.696699	0.723698	0.675814	0.709165	0.682964
$\alpha = \mathbf{10}$	0.688319	0.729412	0.685111	0.713579	0.663306	0.696883	0.664693
$\alpha = \mathbf{20}$	0.668653	0.700691	0.648802	0.677524	0.614724	0.650798	0.595545
$\alpha = \mathbf{30}$	0.663651	0.693924	0.640996	0.670478	0.605531	0.639959	0.583053
$\alpha = \mathbf{40}$	0.660772	0.691046	0.637872	0.667504	0.602163	0.636674	0.578645
$\alpha = \mathbf{50}$	0.65943	0.689444	0.636246	0.666404	0.60056	0.635238	0.576823
$\alpha = \mathbf{60}$	0.658432	0.688849	0.635403	0.66581	0.599142	0.634257	0.575432
$\alpha = \mathbf{70}$	0.657831	0.688425	0.634893	0.665391	0.598497	0.633749	0.57483
$\alpha = \mathbf{80}$	0.657283	0.6882	0.634572	0.665172	0.59808	0.633143	0.574501
$\alpha = \mathbf{90}$	0.656874	0.687997	0.634294	0.664936	0.597839	0.632637	0.574192
$\alpha = \mathbf{100}$	0.655963	0.687767	0.634214	0.664901	0.597719	0.632636	0.573859
$\alpha \to \infty$	0.652587	0.685688	0.63173	0.662879	0.595149	0.629928	0.571167

Table 2: Spearman correlations between Rényi's Squeeziness and PColl.

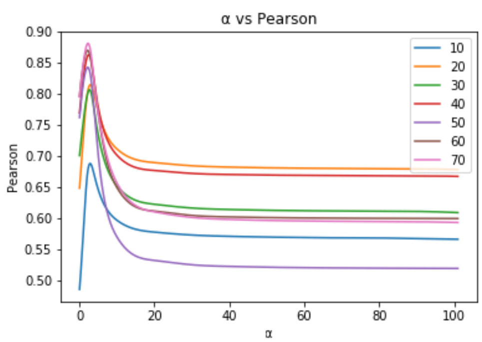

Figure 3: Interpolation of the Pearson correlations.

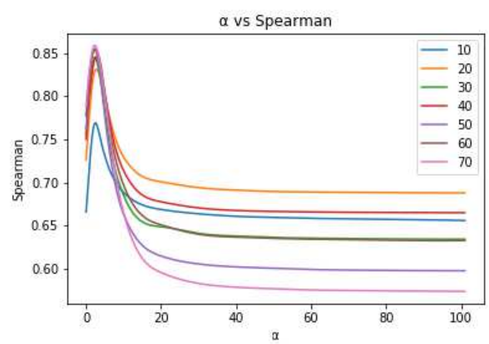

Figure 4: Interpolation of the Spearman correlations.

the correlations that those FSMs will have for an $\alpha \in [0, 101]$. The interpolation of the Pearson correlations is displayed in Figure 3 and the one corresponding to the Spearman correlations is displayed in Figure 4, with each curve corresponding to the different number of states of the FSMs.

All the curves have a peak in the range $(2, 4)$. Therefore, we decided to reproduce the experiment but using $\alpha \in \{2, 2.1, 2.2, 2.3, 2.4, 2.5, 2.6, 2.7, e, 2.8, 2.9, 3, 3.1, \pi, 3.2, 3.3, 3.4, 3.5, 3.6, 3.7, 3.8, 3.9, 4\}$. Then, we obtained the interpolations presented in Figures 5 and 6. From the plots we can observe that, for each number of states, we have a different α that gives the highest correlation but we can bound these values in the interval $(2, 3)$.

As a safety check, we decided to explore if the results are similar in a *real* scenario. In order to do that, we performed our experiments over a recently collected benchmark [27]. This benchmark has 241 deterministic FSMs, which represent real systems (the previously mentioned Set3). We took those FSMs and repeated the experiment: we computed, for each $\alpha \in \{0, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 20, 30, 40, 50, 60, 70, 80, 90, 100, \infty\}$, the values of $\mathtt{Sq}_{\alpha,3}(M)$ and $\mathtt{PColl}_3(M)$ for each FSM M. Then, we computed the Pearson and Spearman correlations between these values. With these correlations, we interpolated the correlations for $\alpha \in [0, 101]$. Due to space limitations, we do not present the results in the paper, but the conclusion is that the interpolation curves behave similarly to the curves that we obtained with the randomly generated FSMs, that is, we obtain again peaks in the values of correlation for values of α belonging to the interval $(2, 3)$. Therefore, we have an empirical

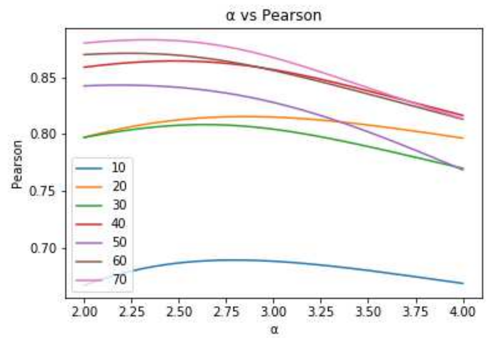

Figure 5: Interpolation of the Pearson correlations at their peak.

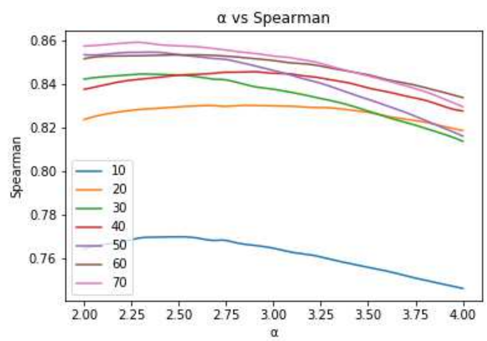

Figure 6: Interpolation of the Spearman correlations at their peak.

confirmation that the results obtained for randomly generated FSMs can be extrapolated to real FSMs.

4.3 Answers to the research questions

As a recap of all the results that we obtained from our experiments, we can answer the Research Questions we performed at the beginning of this section.

Research Question 1. *Does there exist $\alpha \in R_+ \setminus \{1\}$ whose corresponding Squeeziness correlates better with* FEP *than $\alpha = 1$? Is it unique?*

The answer to this question is positive. There exist values of α whose corresponding Squeeziness are better suited to detect FEP than $\alpha = 1$. However, this value is not unique. Although in all our experiments we can bound this α in the interval $(2, 3)$, the actual peak depends on the specific FSM. In any case, we conclude that using Renyi's entropy with values of α in this interval produces better notions of Squeeziness than the original notion where Shannon's entropy was used.

Research Question 2. *Is there an improvement in the capability of Squeeziness to detect cases of* FEP *when the size of the* FSM *increases?*

The answer to this question is that, in general, there is an improvement. However, this improvement is not continuous and sometimes there is a deterioration of the results.

4.4 Threats to validity

We have to explore the threats to internal (related to uncontrolled factors), external (related to generalisation factors) and construct (related to reality factors) validity.

The main threat to the internal validity of our work is associated with the possible faults in the developed tools, which could lead to misleading results. In order to reduce the impact of this threat we tested our code with carefully constructed examples for which we could manually check the results. Luckily, once the FSMs have been (randomly) generated, our experiments had no randomisation factor involved. Therefore, there is no need to repeat the experiments.

The main external validity threat is the different representations of black-box components FSMs. Such a threat cannot be entirely addressed since this population is unknown and it is not possible to sample from an unknown population. In order to reduce the impact of this threat, we used a large number of randomly generated FSMs and checked our results with the results of repeating the experiment in a set of benchmark FSMs that represent real systems.

The main threat to the construct validity of our work is whether the FSMs used in the experiments correspond to possible system components. In order to reduce the impact of this threat, we restricted our range of FSM samples to connected deterministic machines. Also, we checked our results with the results of repeating the experiment in the set of already mentioned benchmark FSMs.

Finally, we have computed (many) results for $\alpha \in [0, 100]$ and have concluded that the peak of the correlations always belongs to the interval $(2, 3)$. Actually, all the curves were strictly decreasing from $\alpha = 3$. However, and this is an important threat to our results, we cannot claim that for a certain size of the analysed FSMs, there do not exist $\alpha \in (100, \infty)$ producing better correlations. We were sampling different values of α in the interval $(100, \infty)$ and confirmed that the correlations were decreasing. We have a strong confidence in this trend but it is not possible to prove that there does not exist a better correlation for a value (or values) of $\alpha \in (100, \infty)$. Note that even if this value exists, but we claim again that this is very unlikely, our experimental results show that it will be difficult to compute sensible correlations. In fact, the results of the Pearson correlation for $\alpha = 100$ already show that five out of seven correlations could not be computed (see penultimate row of Table 1). Therefore, the potential small gain would be mitigated by the problems associated with the computation of the measure.

5 CONCLUSIONS AND FUTURE WORK

It is known that FEP can have a significant effect on testing. Recent work has shown that an information theoretic measure called Squeeziness strongly correlates with the likelihood of FEP both in white-box [11] and black-box [21] scenarios. However, this work only considered Squeeziness based on Shannon's entropy. In this paper we adapted the Squeeziness measure to be based in a more general notion: Rényi's entropy.

Once we defined our new notion of Squeeziness, we carried out experiments in order to evaluate this measure. In the experiments, we compared our measure with PColl, a measure that has been shown to be very good in estimating the likelihood of FEP. We observed a strong correlation between PColl and our notion(s) of Squeeziness (formally, one notion for each $\alpha \in \mathbf{R}_+ \cup \{\infty\}$). Also, we observed better results when we chose values of α belonging to $(2, 3)$. In particular, all these values return better correlations than $\alpha = 1$. Interestingly, our experiments also showed an improvement of the correlations when we were increasing the number of states of the generated FSMs and kept the value of α constant.

For future work, we have several lines of research. We plan to explore approximations, most likely based on sampling, and the trade-off between the cost of sampling (sample size) and the effectiveness of the estimates. We will extend our tool GPTSG [20] with the different pieces of software that we have developed to perform our experiments. Among other features, the tool will automatically choose an a priori *very good* value of α by taking into account the characteristics of the models. The new tool will also generate and process big amounts of mutants [8, 13–15]. Finally, we would like to take previous research as initial step to generalise the framework and measures to deal with asynchronous [17, 24, 25], distributed [7, 18, 19] and cloud [4, 5] systems.

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