Squeeziness for Non-Deterministic Systems☆

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Abstract

Context: Failed Error Propagation greatly reduces the effectiveness of Software Testing by masking faults present in the code. This situation happens when the System Under Test executes a faulty statement, the state of the system is affected by this fault, but the expected output is observed. Therefore, it is a must to assess its impact in the testing process. Squeeziness has been shown to be a useful measure to assess the likelihood of fault masking in deterministic systems.
Objective: The main goal of this paper is to define a new Squeeziness notion that can be used in a scenario where we may have non-deterministic behaviours. The new notion should be a conservative extension of the previous one. In addition, it would be necessary to evaluate whether the new notion appropriately estimates the likelihood that a component of a system introduces Failed Error Propagation.
Method: We defined our black-box scenario where non-deterministic behaviours might appear. Next, we presented a new Squeeziness notion that can be used in this scenario. Finally, we carried out different experiments to evaluate the usefulness of our proposal as an approapriate estimation of the likelihood of Failed Error Propagation.
Results: We found a high correlation between our new Squeeziness notion and the likelihood of Failed Error Propagation in non-deterministic systems. We also found that the extra computation time with respect to the deterministic version of Squeeziness was negligible.
Conclusion: Our new Squeeziness notion is a good measure to estimate the likelihood of Failed Error Propagation being introduced by a component of a system (potentially) showing non-deterministic behaviours. Since it is a conservative extension of the original notion and the extra computation time needed to compute it, with respect to the time needed to compute the former notion, is very small, we conclude that the new notion can be safely used to assess the likelihood of fault masking in deterministic systems.

Keywords: Software Testing, Failed Error Propagation, Non-Deterministic Systems, Information Theory.

1. Introduction

Failed Error Propagation (`FEP`) [1] strongly diminishes the effectiveness of Software Testing [2, 3]. In terms of the RIPR model [4], `FEP` happens if a fault is *reached*, *infecting* the internal state of the system so that it includes some erroneous values, but the error is either not *propagated* outside the system or, even if it is propagated, it is not *revealed* to an external observer. It may be thought that faults causing `FEP` are harmless because they do not have any effect outside the scope where they are located. This false confidence is very dangerous because a slight change in the environment can cause an unforeseen propagation of the error. For example, we may have a system including a faulty component but such that the fault is not revealed due to the current *connections* between the components. However, if we plug this apparently correct but faulty component in another system, then we can have a failure of the whole system.

`FEP` has been widely addressed in the literature. Several studies have found that it really hampers the testing process [5, 6]: in 13% of the examined programs, a total of 60% or more of the tests suffered from `FEP` [7]. The main problem with `FEP` is that it is very difficult to detect. Still, we can try to assess the likelihood of `FEP` by devising measures that could estimate which parts of a system are more likely to suffer from `FEP`. In this line, Information Theory concepts were used to indirectly estimate `FEP`, giving raise to the concept of Squeeziness [8]. This concept, however, was limited to deterministic systems. The effectiveness of

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Squeeziness was shown in an empirical study [9] where 30 programs and more than $7 \cdot 10^6$ tests were used to show that Spearman rank correlation of Squeeziness with FEP is close to 0.95. Subsequent work studied extensions of the Squeeziness notion to new scenarios. In particular, Squeeziness has been adapted to work in a black-box testing framework [10]. Also, the original notion has been slightly modified, introducing Normalised Squeeziness [11], with the goal of comparing Squeeziness of different programs with different input and output sets sizes. The classical notion of Squeeziness is based on Shannon's entropy [12], but there is also work extending it to use other definitions of entropy. Specifically, Rényi's entropy [13] was used to extend Squeeziness by proposing the concept of Rényi's Squeeziness [14]. In addition, a tool supports the choice of a *good* parameter to instantiate Rényi's entropy in an automatic way [15]. There is also recent work extending Squeeziness to develop fine grain versions [16] but, again, it is only suited for white-box scenarios. To the best of our knowledge, Failed Error Propagation has not been studied in the context of systems presenting non-deterministic behaviours. In particular, a notion based on Squeeziness has not been used in non-deterministic systems as an indicator of Failed Error Propagation. Since Squeeziness can be used to assess the likelihood of having cases of FEP in a System Under Test (SUT), testers may have a reference on how easy/hard will be to test this SUT. This is important when planing a testing process, as SUTs with higher Squeeziness will need more effort devoted to testing.

The main limitation of Squeeziness is that it is intrinsically associated with deterministic systems. Although this is a reasonable assumption if we consider testing an isolated component, more complex systems usually present some kind of non-determinism. For example, we may conform a system as the composition of a set of communicating components. Even if all the components were deterministic, an external observer might see non-deterministic behaviours. If we are testing in a black-box framework, then we can only apply inputs and observe outputs, without having access to neither the structure nor the internal communications between components. This framework is illustrated in Figure 1 and it is the one that we consider in this paper. We have a single component C' (right hand side) and this component receives a sequence of values from another component C (left hand side, C can be the composition of several sub-components). Note that this sequence plays a double role: it is an input sequence received by C' and an output sequence produced by C. Since we consider a black-box framework, the tester cannot observe this sequence: the tester provides (input) sequences to the whole system and observes (output) sequences. FEP happens if C provides an unexpected sequence but C' produces the same output sequence for both the expected and unexpected sequences. It is important to remark that, due to this black-box framework, the components cannot be separated and tested in isolation. In order to represent our systems, we use the well-known, and widely used in model-based testing, Finite State Machine (FSM) formalism. However, we can easily adapt our framework to deal with other state-based formalisms as long as they provide a mechanism to apply inputs and observe outputs.

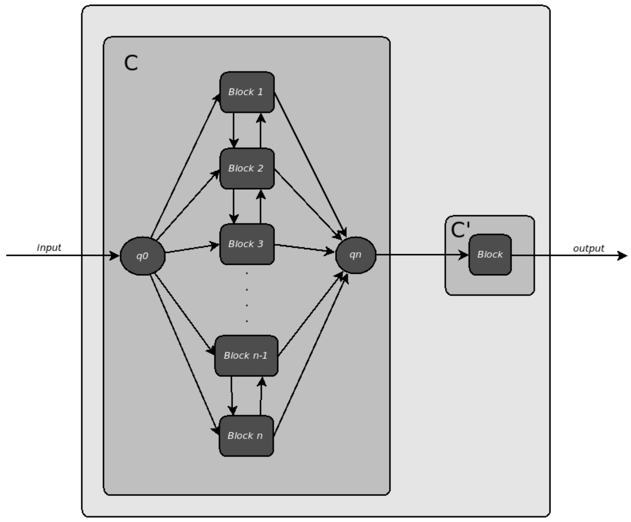

Figure 1: System with non-deterministic behaviour

Squeeziness is defined as the difference between the entropies of inputs and outputs. In a deterministic system, if all the inputs produce different outputs then Squeeziness is equal to 0 and we know that we are free of FEP. In fact, as we will discuss later, FEP can come only from *collisions* of different inputs producing the same output. However, non-deterministic behaviours do not fit this simple pattern because we also need to take into account that the same input can produce different outputs. It is important to emphasise that this new characteristic must be carefully included in the new definition of Squeeziness: the *diffusion* generated by an input being able to produce different outputs should increase the value of the new measure.

Since we are considering a black-box framework and Squeeziness strongly depends on the inputs and outputs that the SUT may produce, we need to provide a method to compute them. If we have a specification of the system, as it is usually the case in the context of formal testing [17, 18], and we assume the *competent programmer hypothesis* [19] then we may consider that the SUT will be very similar to the specification. Therefore, if we use as input and output sets the ones provided by the specification we will have a very accurate description of the ones conforming the SUT. If we do not have a specification, then we might collect sequences during testing, applying inputs and observing outputs, and "infer" these sets according to the obtained sequences. Note that if a certain test suite is not covering a part of the SUT, then we cannot have information about the sequences produced when traversing that part of the system. This would not be a drawback of our approach but of the test suite itself.

Once we were able to produce a satisfactory definition of the new notion, having the *expected* properties, we had to evaluate whether it correlated with the likelihood of FEP. We performed two groups of experiments. First, we created *simulations* of systems by generating inputs and their corresponding outputs (we do not use the internal structure of the system). These experiments aimed to assess the suitability of our approach and its scalability. The results were very satisfactory because correlations were higher than 0.7 in more that 96% of the subjects and higher than 0.9 in more than 88% of the subjects. Second, we evaluated correlation in FSMs, extracted from a well-known benchmark, with the goal of assessing the performance of our approach over FSMs. Since these FSMs were deterministic, we followed a methodology used in recent work [20] to produce non-deterministic versions of them. In this case, we got correlations higher than 0.9 in most of the cases, with only one case under that mark (but still over 0.5).

The rest of the paper is structured as follows. In Section 2 we present the theoretical background needed to define non-deterministic Squeeziness, while its formal definition is given in Section 3. In Section 4 we present our research questions and the experiments that we performed to answer them. In Section 5 we discuss some of the decisions underlying the definition and evaluation of non-deterministic Squeeziness. Section 6 analyses the threats to the validity of our results. Finally, in Section 7 we present our conclusions and some lines for future work.

2. Theoretical Background

In this section we review concepts that we will use during the rest of the paper. *Finite State Machines* (FSMs) will be used to define our experimental subjects. The deterministic version of *Squeeziness* will be the base to define its non-deterministic counterpart. The *probability of collisions* measures the likelihood of Failed Error Propagation (FEP) due to colliding outputs. Note that colliding outputs are the only potential source of FEP in deterministic systems. In addition, we will introduce the *probability of diffusion* to measure the likelihood of having FEP due to diffused inputs. Finally, we will explain how we are going to estimate the likelihood of FEP in a non-deterministic system.

2.1. Finite State Machines

Finite State Machines have been widely used in formal approaches to testing [21]. Here we present some concepts taken from classical work. Although these concepts are based on original sources, some notation is adapted to facilitate the formulation of subsequent definitions.

Given a finite set A, A^* denotes the set of finite sequences of elements of A; A^+ denotes the set of non-empty finite sequences of elements of A; and $\epsilon \in A^*$ denotes the empty sequence. We let $|A|$ denote the size of A. Given a sequence $\tau \in A^*$, $|\tau|$ denotes its length. Given a sequence $\tau \in A^*$ and $a \in A$, we have that τa denotes the sequence τ followed by a and $a\tau$ denotes the sequence τ preceded by a.

An FSM is a finite labelled transition system in which each transition has a label in the form of an *input/output pair*.

Definition 1. *A* Finite State Machine *(FSM) is represented by a tuple $M = (Q, q_{in}, I, O, T)$ in which Q is a finite set of states, $q_{in} \in Q$ is the initial state, I is a finite set of input actions, O is a finite set of output actions, and $T \subseteq Q \times (I \times O) \times Q$ is the transition relation. The meaning of a* transition *$(q, (i, o), q') \in T$, also denoted by $(q, i/o, q')$, is that if M receives the input action i when in state q then it can move to state q' and produce the output action o.*

We say that M is deterministic *if for all $q \in Q$ and $i \in I$ there exists at most one pair $(q', o) \in Q \times O$ such that $(q, i/o, q') \in T$; otherwise, we say that M is* non-deterministic. *We say that M is* observable *if for each $q \in Q$ and $i \in I$ there do not exist $o \in O$ and different $q', q'' \in Q$ such that $(q, i/o, q'), (q, i/o, q'') \in T$. We say that M is* complete *if for each $q \in Q$ and $i \in I$ there exist $o \in O$ and $q' \in Q$ such that $(q, i/o, q') \in T$.*

We assume that FSMs can be non-deterministic. Obviously, deterministic FSMs are a subset of non-deterministic ones. We restrict our attention to observable FSMs since it is easy to transform any non-deterministic FSM into an equivalent observable, probably non-deterministic, FSM. We allow *partial* FSMs, that is, we do not force that our systems are able to accept all the inputs in all their states. By allowing partial and non-deterministic FSMs, we are able to deal with a big variety of systems that are usually omitted in approaches to testing from FSMs. In addition, this increases the applicability of the devised measure as a tool to assess the likelihood of systems having FEP.

A process can be identified with its initial state and we can define a process corresponding to a state q of M by making q the initial state. Thus, we use states and processes and their notation interchangeably. In our work we also assume the *minimal test hypothesis* [22]: the SUT can be modelled as an (unknown) object described in the same formalism as the specification (here, an FSM). It is important to remark that we are in a black-box framework. Therefore, we can only assume the existence of such an FSM, but we might not have access to its description. Furthermore, we can weaken this assumption to only consider that for each input sequence applied to the SUT it will return an output sequence of the same size. If the SUT cannot process the applied input sequence, then we assume that an error is produced. We depict FSMs as diagrams in which nodes represent the states of the FSM, arcs represent transitions between states, and the initial state is denoted by an incoming edge with no source.

In our setting, we distinguish between input actions and *inputs* of the system: an input action is a single element of I while an input of the system will be an element of I^+, that is, a non-empty sequence of input actions (similarly for outputs and output actions). In order to compute Squeeziness, we need to obtain the inputs and outputs of the `FSM` and we do this by using the concept of *trace*.

Definition 2. *Let $M = (Q, q_{in}, I, O, T)$ be an `FSM`. We use the following notation.*

1. *Let $\tau = (i_1, o_1) \dots (i_k, o_k) \in (I \times O)^*$ be a sequence of input/output actions and q be a state. We say that M can perform τ from q if there exist states $q_1 \dots q_k \in Q$ such that for all $1 \leq j \leq k$ we have $(q_{j-1}, i_j/o_j, q_j) \in T$, where $q_0 = q$. We denote this by either $q \xRightarrow{\tau} q_k$ or $q \xRightarrow{\tau}$. If $q = q_{in}$ then we say that τ is a* trace *of M. We denote by $\mathtt{traces}(M)$ the set of traces of M. Note that for every state q we have that $q \xRightarrow{\epsilon} q$ holds. Therefore, $\epsilon \in \mathtt{traces}(M)$ for every `FSM` M.*
2. *Let $\alpha = i_1 \dots i_k \in I^*$ be a sequence of input actions and q be a state. We define $\mathtt{out}_M(q)\alpha$ as the set*

$$\{o_1 \dots o_k \in O^* | q \xRightarrow{(i_1,o_1)\dots(i_k,o_k)}\}$$

Note that if M is deterministic then this set is either empty or a singleton.

3. *Let $q \in Q$ be a state. We define $\mathtt{dom}_M(q)$ as the set*

$$\{\alpha \in I^* | \mathtt{out}_M(q)\alpha \neq \emptyset\}$$

If $q = q_{in}$ then we simply write $\mathtt{dom}_M$. Similarly, we define $\mathtt{image}_M(q)$ as the set

$$\{o_1 \dots o_k \in O^* | \exists i_1 \dots i_k \in I^* : q \xRightarrow{(i_1,o_1)\dots(i_k,o_k)}\}$$

If $q = q_{in}$ then we simply write $\mathtt{image}_M$. We denote by $\mathtt{dom}_{M,k}$ the set $\mathtt{dom}_M \cap I^k$. Similarly, we denote by $\mathtt{image}_{M,k}$ the set $\mathtt{image}_M \cap O^k$.

4. *We define $f_M : \mathtt{dom}_M \longrightarrow \mathcal{P}(\mathtt{image}_M)$ as the function such that for all $\alpha \in \mathtt{dom}_M$ we have $f_M(\alpha) = \{\beta \in O^* | \beta \in \mathtt{out}_M(q_{in})\alpha\}$. Note that if M is deterministic then this set is a singleton and we could define $f_M : \mathtt{dom}_M \longrightarrow \mathtt{image}_M$.*
5. *Let $k > 0$. We define $f_{M,k}$ as the function $f_M \cap (I^k \times O^k)$, where f_M denotes the associated set of pairs. Let $\beta \in \mathtt{image}_M$. We define $f_M^{-1}(\beta)$ as $\{\alpha \in I^* | \beta \in f_M(\alpha)\}$.*

Note that if M is a partial `FSM`, then we have that $\mathtt{dom}_M \subset I^*$; if M is complete, then $\mathtt{dom}_M = I^*$.

2.2. Squeeziness for deterministic systems

In previous work [8, 9, 10, 11, 14], Squeeziness was proposed to assess the likelihood of `FEP` in deterministic systems. Squeeziness is an information theoretic concept that measures the loss of information in a system. In our case, this loss is given by the difference between the information comprised in the set of inputs and the one comprised in the set of outputs. In order to measure the amount of information in a set, we use the classical concept of *entropy* [12]. Therefore, Squeeziness was formally defined as the difference between two entropies.

Definition 3. *Let A be a finite set and ξ_A be a random variable over A. We denote by σ_{ξ_A} the probability distribution induced by ξ_A. The* entropy *of the random variable ξ_A, denoted by $\mathcal{H}(\xi_A)$, is defined as:*

$$\mathcal{H}(\xi_A) = -\sum_{a \in A} \sigma_{\xi_A}(a) \cdot \log_2(\sigma_{\xi_A}(a))$$

Let $f : A \longrightarrow B$ be a total function and consider two random variables ξ_A and ξ_B ranging, respectively, over A and B. The Squeeziness *of f, denoted by $\mathtt{Sq}(f)$, is defined as the loss of information after applying f to A, that is, $\mathcal{H}(\xi_A) - \mathcal{H}(\xi_B)$.*

Squeeziness measures the amount of information lost between the sets A and B through the function f. As explained in our previous work [10], `FSM`s can be seen as functions that transform sequences of input actions into sequences of output actions. To properly transform an `FSM` into a function between two random variables, the only requisite we are missing is to represent the input and output sets of the `FSM` as random variables. This is easily achieved by assigning a probability distribution to each set. Specifically, we assign a probability distribution to the sets of input sequences and of output sequences of a specific length k. As explained in our previous work [10], we took this decision because it gives us an incremental procedure to compute a sequence of consecutive values of Squeeziness so that we can analyse how the series is *evolving*. Actually, the input sequence length used will depend on the amount of testing to be carried out since this will determine the lengths of the input sequences that a component is likely to receive. Note also that there is potential to use Squeeziness values, for different input sequence lengths, to choose test cases. We can use these values with the aim of using test cases that minimise the likelihood of `FEP`, that is, this approach provides a way to know, for a given length, whether the probability of having `FEP`, once we have tested all the possible inputs with the given length, will be greater than 0.

In order to apply Squeeziness to deterministic `FSM`s we define two random variables, $\xi_{\mathtt{dom}_{M,k}}$ and $\xi_{\mathtt{image}_{M,k}}$, ranging respectively over $\mathtt{dom}_{M,k}$ and $\mathtt{image}_{M,k}$.

Definition 4. *Let $M = (Q, q_{in}, I, O, T)$ be a deterministic `FSM` and $k > 0$. Let us consider two random variables $\xi_{\mathtt{dom}_{M,k}}$ and $\xi_{\mathtt{image}_{M,k}}$ ranging, respectively, over the domain and image of $f_{M,k}$. The* Squeeziness *of M* at length k *is defined as*

$$\mathtt{Sq}_k(M) = \mathcal{H}(\xi_{\mathtt{dom}_{M,k}}) - \mathcal{H}(\xi_{\mathtt{image}_{M,k}})$$

In a certain sense, the notion of Squeeziness encodes the distribution over input sequences of length k producing the same output sequence.

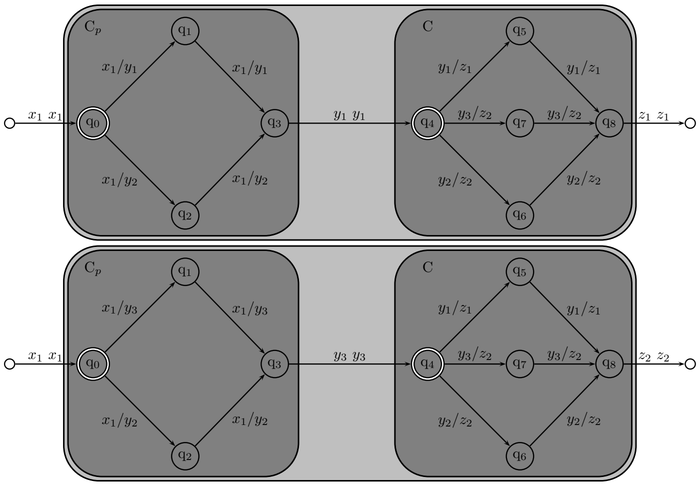

Figure 2: A case of fault masking

2.3. Probability of Collisions

The *probability of collisions* was used in the original definition of Squeeziness [8, 9] to assess FEP. The idea is that each input leading to the same output generates a collision. This collision induces a certain probability of masking a fault because it can lead the system to a state that produces the same output. The original notion assumes a uniform probability over the set of input sequences of the FSM. In this case, the probability of collisions (PColl) can be defined as follows.

Definition 5. *Let M be an FSM and $k > 0$. Let $\mathtt{image}_{M,k} = \{\beta_1, ..., \beta_n\}$ and for all $1 \leq i \leq n$ let $I_i = f^{-1}_{M,k}(\beta_i)$ and $m_i = |f^{-1}_{M,k}(\beta_i)|$. We have that $d = \sum_{i=1}^{n} m_i$ is the size of the input space.*

Given a uniform distribution over the inputs, the probability of α and α' both being in the set I_i is equal to $p_i = \frac{m_i \cdot (m_i - 1)}{d \cdot (d-1)}$. We have that the probability of having a collision in M for sequences of length k, denoted by $\mathtt{PColl}_k(M)$, is given by

$$\mathtt{PColl}_k(M) = \sum_{i=1}^{n} p_i$$

2.4. Probability of Diffusion

Next we introduce a new concept that naturally appears if we are going to assess FEP in non-deterministic systems. The *probability of diffusion* is somehow similar to PColl but it only happens in the presence of non-determinism: generating two or more outputs from the same input diffuses the input to multiple different outputs. Therefore, diffusion induces a certain probability of masking a fault if we reach a state of the system producing one of the different outputs that can be generated by the input. An example of this can be found in Figure 2, where no test can reveal that the second system has a fault because for the same input sequence x_1x_1 the first FSM can obtain both z_1z_1 and z_2z_2, while in the faulty version (the second FSM) we always obtain the second output, that we consider a valid one. However, the internal state between both components has not been a valid internal state, because in the first FSM we cannot have an internal state producing y_3y_3 but this is possible in the second FSM, as shown in Figure 2.

If we assume again, as the original definition did with PColl, a uniform probability over the set of output sequences of the FSM, the probability of diffusion (PDiff) is defined as follows.

Definition 6. *Let M be an FSM and $k > 0$. Let $\mathtt{dom}_{M,k} = \{\alpha_1, ..., \alpha_n\}$ and for all $1 \leq i \leq n$ let $O_i = f_{M,k}(\alpha_i)$ and $m'_i = |f_{M,k}(\alpha_i)|$. We have that $d' = \sum_{i=1}^{n} m'_i$ is the size of the output space.*

Given a uniform distribution over the outputs, the probability of β and β' both being in the set O_i is equal to $p'_i = \frac{m'_i \cdot (m'_i - 1)}{d' \cdot (d'-1)}$, that is, the probability of both outputs being in the output space of the input given the output space of the whole function. Thus, we have that the probability of having a diffusion in M for sequences of length k, denoted by $\mathtt{PDiff}_k(M)$, is given by

$$\mathtt{PDiff}_k(M) = \sum_{i=1}^{n} p'_i$$

The analogy between PColl and PDiff can be seen in their formal definitions: PColl relies on the inverse image of outputs (expressed by setting $m_i = |f^{-1}_{M,k}(\beta_i)|$) while PDiff relies on the size of the image of inputs (expressed by setting $m'_i = |f_{M,k}(\alpha_i)|$). Note that in the deterministic case we have that m'_i is always equal to 1 because $\alpha_i \in \mathtt{dom}_{M,k}$.

2.5. Estimating Failed Error Propagation

In previous work, FEP was estimated by using PColl. This estimation was based on the properties of deterministic systems: the only possible source of fault masking would be the collision of multiple inputs to the same output. In the non-deterministic case we still have this source of fault masking, but it is not the only one. We also have fault masking produced by the *diffusion* of the same input to multiple outputs. We estimate this fault masking through the PDiff measure. Then, in order to estimate the Failed Error Propagation of a non-deterministic system we will use the sum of both phenomena, that is, we will consider PColl+PDiff.

3. Non-Deterministic Squeeziness

In this section we extend Squeeziness to deal with non-deterministic systems. In order to address this task we need to consider two factors: the inclusion of a new source of FEP (given by non-deterministic transitions) and the limitation of the original notion to consider only one source of FEP.

Non-determinism hinders the testing process because the SUT can return different outputs after different applications of the same input. This could produce that a specific erroneous behaviour is not observed due to the fault being masked by the non-determinism of the system. Therefore, even if we apply an input reaching the fault, it may happen that posterior non-determinism leads the component through a path such that the returned output does not reveal the fault.

The second factor complicates things more than expected. Squeeziness was introduced to consider fault masking produced by the collision of several inputs to the same output. Thus, it could be thought that it could deal with the source of FEP produced by the non-determinism. However, the original formulation of Squeeziness is suited to deterministic systems and, therefore, we need to derive again the formula from its high level definition. We start with Definition 4. Using the following result [10]

$$\begin{array}{c}\mathcal{H}(\xi_{\mathtt{image}_{M,k}}) + \mathcal{H}(\xi_{\mathtt{dom}_{M,k}}|\xi_{\mathtt{image}_{M,k}}) \\ \| \\ \mathcal{H}(\xi_{\mathtt{dom}_{M,k}}) + \mathcal{H}(\xi_{\mathtt{image}_{M,k}}|\xi_{\mathtt{dom}_{M,k}})\end{array} \quad (1)$$

we can rewrite Squeeziness as

$$\mathtt{Sq}_k(M) = \mathcal{H}(\xi_{\mathtt{dom}_{M,k}}|\xi_{\mathtt{image}_{M,k}}) - \mathcal{H}(\xi_{\mathtt{image}_{M,k}}|\xi_{\mathtt{dom}_{M,k}})$$

and using some auxiliary results [10] and the fact that

$$\sigma_{\xi_{\mathtt{image}_{M,k}}}(\beta) = \sum_{\alpha \in f_M^{-1}(\beta)} \sigma_{\xi_{\mathtt{dom}_{M,k}}}(\alpha)$$

we can finally rewrite Squeeziness as

$$\mathtt{Sq}_k(M) = -\sum_{\beta \in \mathtt{image}_{M,k}} \left(\sum_{\alpha \in f_M^{-1}(\beta)} \sigma_{\xi_{\mathtt{dom}_{M,k}}}(\alpha) \right) \cdot \mathcal{R}_M(\beta) + \mathcal{S}_M \quad (2)$$

where the term $\mathcal{R}_M(\beta)$ is equal to

$$\sum_{\alpha \in f_M^{-1}(\beta)} \frac{\sigma_{\xi_{\mathtt{dom}_{M,k}}}(\alpha)}{\sigma_{\xi_{\mathtt{dom}_{M,k}}}(f_M^{-1}(\beta))} \cdot \log_2 \left(\frac{\sigma_{\xi_{\mathtt{dom}_{M,k}}}(\alpha)}{\sigma_{\xi_{\mathtt{dom}_{M,k}}}(f_M^{-1}(\beta))} \right) \quad (3)$$

and the term $\mathcal{S}_M$ is given in Figure 3. The full derivation of this formula can be found in Appendix A.

In our previous work [10], we provided the following equivalent formulation of Squeeziness

$$\mathtt{Sq}_k(M) = -\sum_{\beta \in \mathtt{image}_{M,k}} \left(\sum_{\alpha \in f_M^{-1}(\beta)} \sigma_{\xi_{\mathtt{dom}_{M,k}}}(\alpha) \right) \cdot \mathcal{R}_M(\beta)$$

because $\mathcal{S}_M$ is equal to 0 if the FSM is deterministic. However, this formulation is no longer valid if we are in a non-deterministic scenario because, in general, $\mathcal{S}_M \neq 0$. Then, it is necessary to start from Equation 2.

In that equation, the term $\mathcal{S}_M$ accounts for the increment of information introduced by non-determinism, reducing the total loss of information of the FSM. This term will be relevant on the development of non-deterministic Squeeziness.

The previous formulation allows us to measure the loss of information produced by the FSM. However, as explained before, in this scenario the loss of information is not the only source of FEP. The effect of non-determinism is that an increase in information generates FEP as well. Therefore, we also need to measure the increment of information produced by the FSM and the more natural way to do it is by using the new notion of *Alternative* Squeeziness.

Definition 7. *Let $M = (Q, q_{in}, I, O, T)$ be an FSM and $k > 0$. Let us consider two random variables $\xi_{\mathtt{dom}_{M,k}}$ and $\xi_{\mathtt{image}_{M,k}}$ ranging, respectively, over the domain and image of $f_{M,k}$. The Alternative Squeeziness of M at length k is defined as*

$$\mathtt{AlSq}_k(M) = \mathcal{H}(\xi_{\mathtt{image}_{M,k}}) - \mathcal{H}(\xi_{\mathtt{dom}_{M,k}})$$

The following result, where we provide an alternative formulation of Alternative Squeeziness, is a straightforward consequence of the previous definition by using the following fact:

$$\sigma_{\xi_{\mathtt{dom}_{M,k}}}(\alpha) = \sum_{\beta \in f_M(\alpha)} \sigma_{\xi_{\mathtt{image}_{M,k}}}(\beta)$$

Corollary 1. *Let $M = (Q, q_{in}, I, O, T)$ be an FSM and $k > 0$. Let us consider a random variable $\xi_{\mathtt{image}_{M,k}}$ ranging over the image of $f_{M,k}$. We have that*

$$\mathtt{AlSq}_k(M) = -\sum_{\alpha \in \mathtt{dom}_{M,k}} \left(\sum_{\beta \in f_M(\alpha)} \sigma_{\xi_{\mathtt{image}_{M,k}}}(\beta) \right) \cdot \mathcal{R}'_M(\alpha) + \mathcal{S}'_M$$

where the term $\mathcal{R}'_M(\alpha)$ is equal to

$$\sum_{\beta \in f_M(\alpha)} \frac{\sigma_{\xi_{\mathtt{image}_{M,k}}}(\beta)}{\sigma_{\xi_{\mathtt{image}_{M,k}}}(f_M(\alpha))} \cdot \log_2 \left(\frac{\sigma_{\xi_{\mathtt{image}_{M,k}}}(\beta)}{\sigma_{\xi_{\mathtt{image}_{M,k}}}(f_M(\alpha))} \right) \quad (4)$$

and the term $\mathcal{S}'_M$ is given in Figure 3.

The proof of the previous result can be found in Appendix B. In this formulation, similarly to what happened with the formula of Squeeziness, the term $\mathcal{S}'_M$ accounts for the decrease on information produced by the collision of multiple inputs to the same output, reducing the total gain of information of the FSM. This factor is equal to 0 when there is no possible loss of information, that is, when each output is produced by only one input. This term will also be relevant when defining the non-deterministic extension of Squeeziness.

It is easy to see that Squeeziness and Alternative Squeeziness have the same absolute value but with different sign. Obviously, given an FSM M and a length k, if we add those

$$\mathcal{S}_M = \sum_{\alpha \in \mathtt{dom}_{M,k}} \sigma_{\xi_{\mathtt{dom}_{M,k}}}(\alpha) \cdot \sum_{\beta \in f_M(\alpha)} \frac{\sum_{\alpha \in f_M^{-1}(\beta)} \sigma_{\xi_{\mathtt{dom}_{M,k}}}(\alpha)}{\sum_{\beta \in f_M(\alpha)} \sum_{\alpha \in f_M^{-1}(\beta)} \sigma_{\xi_{\mathtt{dom}_{M,k}}}(\alpha)} \cdot \log_2 \left(\frac{\sum_{\alpha \in f_M^{-1}(\beta)} \sigma_{\xi_{\mathtt{dom}_{M,k}}}(\alpha)}{\sum_{\beta \in f_M(\alpha)} \sum_{\alpha \in f_M^{-1}(\beta)} \sigma_{\xi_{\mathtt{dom}_{M,k}}}(\alpha)} \right)$$

$$\mathcal{S}'_M = \sum_{\beta \in \mathtt{image}_{M,k}} \sigma_{\xi_{\mathtt{image}_{M,k}}}(\beta) \cdot \sum_{\alpha \in f_M^{-1}(\beta)} \frac{\sum_{\beta \in f_M(\alpha)} \sigma_{\xi_{\mathtt{image}_{M,k}}}(\beta)}{\sum_{\alpha \in f_M^{-1}(\beta)} \sum_{\beta \in f_M(\alpha)} \sigma_{\xi_{\mathtt{image}_{M,k}}}(\beta)} \cdot \log_2 \left(\frac{\sum_{\beta \in f_M(\alpha)} \sigma_{\xi_{\mathtt{image}_{M,k}}}(\beta)}{\sum_{\alpha \in f_M^{-1}(\beta)} \sum_{\beta \in f_M(\alpha)} \sigma_{\xi_{\mathtt{image}_{M,k}}}(\beta)} \right)$$

Figure 3: Definition of $\mathcal{S}_M$ (top) and $\mathcal{S}'_M$ (bottom)

values as an attempt to estimate the total amount of FEP in M for a given length k, we would obtain 0. However, if we analyse the formulas, we can conclude that the correction factors (S_M and S'_M) are diminishing the effect of the source of FEP we are evaluating. Therefore, the original formulas are not a viable solution to compute the likelihood of FEP.

In fact, we do not need to evaluate whether the FSM loses or creates information in absolute terms. Instead, we need to know how much information can be potentially lost or gained. Specifically, we would like to obtain the maximum information that the FSM can lose, so that we can account for the possible produced collisions, and the maximum information that the FSM can generate, so that we can also account for the possible produced diffusion. Thus, the solution we propose is to remove the correction factors $\mathcal{S}_M$ and $\mathcal{S}'_M$ from the formulas. This way, we can consider the maximum possible effect of both sources of FEP. We further discuss this decision in Section 5.

Taking into account the previous considerations, we finally formulate Non-Deterministic Squeeziness as follows.

Definition 8. *Let $M = (Q, q_{in}, I, O, T)$ be an FSM and $k > 0$. Let us consider two random variables $\xi_{\mathtt{dom}_{M,k}}$ and $\xi_{\mathtt{image}_{M,k}}$ ranging , respectively, over the domain and image of $f_{M,k}$. We have that*

$$\begin{aligned}\mathtt{NDSq}_k(M) = &-\sum_{\beta \in \mathtt{image}_{M,k}} \left(\sum_{\alpha \in f_M^{-1}(\beta)} \sigma_{\xi_{\mathtt{dom}_{M,k}}}(\alpha) \right) \cdot \mathcal{R}_M(\beta) \\ &-\sum_{\alpha \in \mathtt{dom}_{M,k}} \left(\sum_{\beta \in f_M(\alpha)} \sigma_{\xi_{\mathtt{image}_{M,k}}}(\beta) \right) \cdot \mathcal{R}'_M(\alpha)\end{aligned}$$

where the terms $\mathcal{R}_M(\beta)$ and $\mathcal{R}'_M(\alpha)$ are given in Equations 3 and 4, respectively.

In the next section we will evaluate the usefulness of Non-Deterministic Squeeziness to assess the likelihood of FEP in non-deterministic systems. First, we have to consider the probability distributions that we assign to the random variables associated with the domain and the image of the FSM. If we knew the true probability distribution, then such distribution would be the ideal one. However, in most situations, we do not even have a hint on the distribution and it is necessary to make an assumption. This was also the case in previous work dealing with (deterministic) Squeeziness. However, unlike in previous work, we need to define not only a distribution for the inputs, but also one for the outputs of the FSM.

This situation raises another concern: should we define one distribution and bound the other one to this first one, or should we consider the same distribution for both sets, independently of the relations that the FSM could generate? The first case would be something to consider, as the distribution over the outputs is influenced by the distribution over the inputs and the mapping of inputs to outputs. However, after some preliminary experiments, we have chosen the second approach because the first one showed substantially worse empirical results. We think that this situation happens because we do not know the real distributions either for inputs or outputs, although further work would be needed to show more evidence. Also, it is important to remark that in a black-box situation we would not be able to derive the outputs distribution from the inputs one. In order to make the assumption of probability distributions, and following previous work on Squeeziness, we analyse two approaches.

3.1. Maximum Entropy Principle

In this approach we choose a distribution that maximises the entropy. In our setting, this distribution is the uniform distribution [23]. Under this assumption, the weight of a single element of $\sigma_{\xi_{\mathtt{dom}_{M,k}}}$ would be $\frac{1}{|\mathtt{dom}_{M,k}|}$ and of $\sigma_{\xi_{\mathtt{image}_{M,k}}}$ would be $\frac{1}{|\mathtt{image}_{M,k}|}$. Thus, the weight of the inverse image of an output $\beta \in \mathtt{image}_{M,k}$ would be equal to $\frac{|f_M^{-1}(\beta)|}{|\mathtt{dom}_{M,k}|}$ and the weight of the image of an input $\alpha \in \mathtt{dom}_{M,k}$ would be equal to $\frac{|f_M(\alpha)|}{|\mathtt{image}_{M,k}|}$.

After applying these definitions, the formula of Non-Deterministic Squeeziness can be found in Figure 4 (top). Let us mention that we will use this formula in our experiments because we this principle is also followed by the definitions of PColl and PDiff.

$$\begin{aligned}\mathtt{NDSq}_k(M) =& -\sum_{\beta\in\mathtt{image}_{M,k}}\left(\sum_{\alpha\in f_M^{-1}(\beta)}\frac{1}{|\mathtt{dom}_{M,k}|}\right)\cdot\left(\sum_{\alpha\in f_M^{-1}(\beta)}\frac{\frac{1}{|\mathtt{dom}_{M,k}|}}{\frac{|f_M^{-1}(\beta)|}{|\mathtt{dom}_{M,k}|}}\cdot\log_2\left(\frac{\frac{1}{|\mathtt{dom}_{M,k}|}}{\frac{|f_M^{-1}(\beta)|}{|\mathtt{dom}_{M,k}|}}\right)\right)\\ &-\sum_{\alpha\in\mathtt{dom}_{M,k}}\left(\sum_{\beta\in f_M(\alpha)}\frac{1}{|\mathtt{image}_{M,k}|}\right)\cdot\left(\sum_{\beta\in f_M(\alpha)}\frac{\frac{1}{|\mathtt{image}_{M,k}|}}{\frac{|f_M(\alpha)|}{|\mathtt{image}_{M,k}|}}\cdot\log_2\left(\frac{\frac{1}{|\mathtt{image}_{M,k}|}}{\frac{|f_M(\alpha)|}{|\mathtt{image}_{M,k}|}}\right)\right)\\ =& \frac{1}{|\mathtt{dom}_{M,k}|}\cdot\sum_{\beta\in\mathtt{image}_{M,k}}|f_M^{-1}(\beta)|\cdot\log_2(|f_M^{-1}(\beta)|)+\frac{1}{|\mathtt{image}_{M,k}|}\cdot\sum_{\alpha\in\mathtt{dom}_{M,k}}|f_M(\alpha)|\cdot\log_2(|f_M(\alpha)|)\end{aligned}$$

$$\begin{aligned}\mathtt{NDSq}_k(M) =& -\left(\sum_{\alpha\in f_M^{-1}(\beta')}\frac{1}{|f_M^{-1}(\beta')|}\right)\cdot\left(\sum_{\alpha\in f_M^{-1}(\beta')}\frac{1}{|f_M^{-1}(\beta')|}\cdot\log_2\left(\frac{1}{|f_M^{-1}(\beta')|}\right)\right)\\ &-\left(\sum_{\beta\in f_M(\alpha')}\frac{1}{|f_M(\alpha')|}\right)\cdot\left(\sum_{\beta\in f_M(\alpha')}\frac{1}{|f_M(\alpha')|}\cdot\log_2\left(\frac{1}{|f_M(\alpha')|}\right)\right)\\ =& \log_2(|f_M^{-1}(\beta')|)+\log_2(|f_M(\alpha')|)\end{aligned}$$

Figure 4: Definition of $\mathtt{NDSq}_k(M)$ under maximum entropy (top) and under maximum information balance (loss and gain) (bottom)

3.2. Maximum Information Balance (Loss and Gain)

In this approach we look for a distribution that maximises the difference in information produced by the FSM. For the set of inputs, such distribution is the one that is uniformly distributed over the largest inverse image of an element of the outputs and zero elsewhere. This distribution achieves maximum loss of information [8]. Similarly, for the set of outputs, the distribution that achieves the maximum gain of information is the one that is uniformly distributed over the largest image of an input and zero elsewhere.

Formally, consider $\beta' \in \mathtt{image}_{M,k}$ such that for all $\beta \in \mathtt{image}_{M,k}$ we have that $|f_M^{-1}(\beta')| \geq |f_M^{-1}(\beta)|$. Then,

$$\sigma_{\xi_{\mathtt{dom}_{M,k}}}(\alpha) = \begin{cases} \frac{1}{|f_M^{-1}(\beta')|} & \text{if } \alpha \in f_M^{-1}(\beta') \\ 0 & \text{otherwise} \end{cases}$$

Similarly, consider $\alpha' \in \mathtt{dom}_{M,k}$ such that for all $\alpha \in \mathtt{dom}_{M,k}$ we have that $|f_M(\alpha')| \geq |f_M(\alpha)|$. Then,

$$\sigma_{\xi_{\mathtt{image}_{M,k}}}(\beta) = \begin{cases} \frac{1}{|f_M(\alpha')|} & \text{if } \beta \in f_M(\alpha') \\ 0 & \text{otherwise} \end{cases}$$

After using these probability distributions in the generic definition of Non-Deterministic Squeeziness, the formulation can be found in Figure 4 (bottom).

4. Experiments

In order to evaluate the effectiveness of Non-Deterministic Squeeziness to assess FEP, we conducted two types of experiments. The first category includes experiments conducted to preliminary test the proposal and to observe its behaviour in *limit* cases, that is, using small and huge subjects. In the second category we test our proposal on FSMs extracted from a benchmark. The code, results and experimental subjects of our experiments can be found at `https://github.com/Colosu/NDSq`.

First, we will introduce the research questions that we asked ourselves to evaluate the effectiveness of our proposal.

4.1. Research questions

The goal of our work is to properly evaluate the likelihood of FEP in non-deterministic systems. Therefore, our first step is to assess how well Non-Deterministic Squeeziness addresses this problem.

Research Question 1. *Is there a strong correlation between Failed Error Propagation and Non-Deterministic Squeeziness? Is this correlation stable along different situations?*

We would also like to compare our new proposal and our previous one to empirically decide whether we really needed an improved version of Squeeziness.

Research Question 2. *Is the correlation between Failed Error Propagation and Non-Deterministic Squeeziness higher than the correlation between Failed Error Propagation and Squeeziness?*

Additionally, we would like to see how Non-Deterministic Squeeziness performs over FSMs so that we can infer how it will behave when applied to other systems.

Research Question 3. *Is there a correlation between Failed Error Propagation and Non-Deterministic Squeeziness when applied over* `FSMs`*?*

Finally, we would like to explore the performance of Non-Deterministic Squeeziness, compared to the performance of Squeeziness, to check whether there is a high extra computation cost for using the non-deterministic version.

Research Question 4. *Is the extra computation time needed for Non-Deterministic Squeeziness an important performance issue?*

4.2. Experiments I: Simulated systems

Our first set of experiments used simulations so that we could quickly assess the suitability of our approach and explore its behaviour in limit cases. These experiments also provide an estimation on how our proposal could perform for big systems (with up to $100,000$ inputs), in order to test its scalability. We followed the methodology presented in previous work on Squeeziness [8, 10] to perform these experiments.

# Inputs	**Maximum # inputs to same output**	**Maximum # outputs from same input**	**Pearson correlation with Non – Det. Squeeziness**	**Pearson correlation with Det. Squeeziness**
100	10	10	0.8912	0.7374
50000	10	10	0.7217	0.9377
50000	20	10	0.5987	0.5300
50000	20	20	0.6437	0.6470
100000	20	10	0.5163	0.5494
100000	20	20	0.3757	0.6118
100000	50	10	0.4458	−0.3832
100000	50	20	0.5332	−0.3582
100000	100	10	0.9846	0.3869
100000	100	20	0.9825	0.3488

Table 1: Outstanding results of the simulated experiments

The experimental subjects were created at run-time. Our algorithm does not use the internal structure of the `FSM` because it only needs the input and output sets and their association (that is, which inputs produce a certain output). First, we need to set the relevant parameters: number of inputs, the maximum number of inputs that can lead to an individual output, and the maximum number of outputs that can be generated by one input. Using these three values, a set of outputs, with associated number of inputs that lead to each of them, is randomly generated. Next, it is necessary to assign to each input the number of outputs that can be produced by them. Finally, the process of associating each input with their corresponding outputs begins. If we end up with inputs that do not have outputs left to be associated with them, according to the preset bounds, then new outputs are generated so that the process can conclude.

We generated 200 experimental subjects, according to the parameters, and computed the addition of PColl and PDiff and the corresponding Non-Deterministic Squeeziness values. In order to compute Non-Deterministic Squeeziness we applied the Maximum Entropy Principle, thus assuming a uniform probability distribution. Then, we compute Pearson and Spearman correlations between PColl + PDiff and Non-Deterministic Squeeziness. The whole process is repeated 10 times and the mean of these 10 correlation values is presented as the final correlation for the initial parameters.

This experiment is repeated with 178 sets of different parameter values. In Table 1 we enumerate the ten most interesting cases. Specifically, we show all the entries with a correlation lower than 0.7, the two highest values entries, and the first result to have a representation of the entries with small number of inputs. The results can be found in Tables 2, 3 and 4. In these tables we only show Pearson correlation values because Spearman correlation values were almost the same (this is not the case in our second set of experiments). We can observe that the correlations range from 0.3757 to 0.9846, all positive values, and there are only 6 cases, out of 178 entries, with correlation lower than 0.7. Therefore, 96.65% of the experiments returned correlations higher than 0.7. Moreover, 88.83% of them have a correlation higher than 0.9. This implies a strong correlation between the addition of PColl and PDiff, and Non-Deterministic Squeeziness. Moreover, the correlation is very stable for most of the scenarios, with only 6 *irregular* cases. These cases correspond to situations where the number of inputs is much higher than the potential non-determinism. We provide a more detailed discussion about this effect in Section 5.

In order to compare the performance of Squeeziness and Non-Deterministic Squeeziness, we decided to compute Squeeziness at the same time when executing these experiments. Then, we performed a statistical significance test to check whether the results corresponding to Squeeziness and Non-Deterministic Squeeziness were *similar*. To be precise, we explored the null hypothesis that claims that the results provided by both notions follow the same probability distribution. First, we performed an homogeneity of variance check that arose a negative value. Thus, we performed a Kruskal-Wallis H-test that returned a p-value of $7.78 \cdot 10^{-57}$. This represents that there is an almost null probability that the null hypothesis holds. Therefore, we can reject the null hypothesis with a confidence higher than 99% (its p-value is lower than 0.01). In order to double-check our results, we also performed a t-test and obtained a lower p-value (of $1.1 \cdot 10^{-101}$). Thus, the conclusion is that the performance of Non-Deterministic Squeeziness and Squeeziness are not similar and we can conclude that Non-Deterministic Squeeziness is better in our scenario.

4.3. Experiments II: using `FSMs`

In order to evaluate Non-Deterministic Squeeziness over `FSMs` representing systems, we considered experimental subjects extracted from a well-known benchmark, the ACM/SIGDA

# Inputs	**Maximum # inputs to same output**	**Maximum # outputs from same input**	**Pearson Correlation with Non − Det. Squeeziness**	**Pearson Correlation with Det. Squeeziness**
100	10	10	0.8912	0.7374
100	20	10	0.9181	0.4212
100	20	20	0.9377	0.2277
100	50	10	0.9694	0.1075
100	50	20	0.9689	0.0494
100	50	50	0.9682	0.0547
200	10	10	0.9126	0.7488
200	20	10	0.8988	0.5883
200	20	20	0.9244	0.3776
200	50	10	0.9466	0.1441
200	50	20	0.9465	0.0797
200	50	50	0.9503	0.0555
200	100	10	0.9715	0.0937
200	100	20	0.9695	0.0726
200	100	50	0.9708	0.1196
200	100	100	0.9695	0.0675
500	10	10	0.9737	0.7974
500	20	10	0.9063	0.6535
500	20	20	0.9056	0.5045
500	50	10	0.9150	0.4930
500	50	20	0.9419	0.1845
500	50	50	0.9527	0.0879
500	100	10	0.9349	0.2353
500	100	20	0.9535	0.0583
500	100	50	0.9547	0.0252
500	100	100	0.9503	0.0619
500	200	10	0.9659	0.0177
500	200	20	0.9606	0.0077
500	200	50	0.9623	−0.0154
500	200	100	0.9621	−0.0190
500	200	200	0.9599	−0.0026
1000	10	10	0.9736	0.8715
1000	20	10	0.9215	0.6510
1000	20	20	0.9163	0.5674
1000	50	10	0.9274	0.5330
1000	50	20	0.9205	0.3899
1000	50	50	0.9587	0.0580
1000	100	10	0.9266	0.4663
1000	100	20	0.9446	0.1667
1000	100	50	0.9561	0.0529
1000	100	100	0.9522	0.0188
1000	200	10	0.9401	0.2611
1000	200	20	0.9530	−0.0006
1000	200	50	0.9566	−0.0026
1000	200	100	0.9524	−0.0058
1000	200	200	0.9523	−0.0084
1000	500	10	0.9730	0.0881
1000	500	20	0.9699	0.0823
1000	500	50	0.9702	0.0515
1000	500	100	0.9708	0.0950
1000	500	200	0.9715	0.0410
1000	500	500	0.9712	0.0867

Table 2: Results of the simulated experiments (part 1)

# Inputs	**Maximum # inputs to same output**	**Maximum # outputs from same input**	**Pearson Correlation with Non − Det. Squeeziness**	**Pearson Correlation with Det. Squeeziness**
2000	10	10	0.8882	0.9097
2000	20	10	0.9535	0.6630
2000	20	20	0.9278	0.5418
2000	50	10	0.9194	0.5777
2000	50	20	0.9286	0.4059
2000	50	50	0.9303	0.2276
2000	100	10	0.9320	0.5057
2000	100	20	0.9331	0.3188
2000	100	50	0.9581	0.0336
2000	100	100	0.9599	0.0473
2000	200	10	0.9336	0.3896
2000	200	20	0.9437	0.1691
2000	200	50	0.9586	0.0546
2000	200	100	0.9583	−0.0254
2000	200	200	0.9537	0.0378
2000	500	10	0.9492	0.0520
2000	500	20	0.9549	−0.0531
2000	500	50	0.9560	−0.0175
2000	500	100	0.9556	−0.0252
2000	500	200	0.9559	0.0207
2000	500	500	0.9578	−0.0009
5000	10	10	0.7601	0.9125
5000	20	10	0.9789	0.6141
5000	20	20	0.9664	0.5325
5000	50	10	0.9292	0.5693
5000	50	20	0.9368	0.4776
5000	50	50	0.9420	0.3050
5000	100	10	0.9375	0.5718
5000	100	20	0.9370	0.4148
5000	100	50	0.9401	0.2414
5000	100	100	0.9583	0.0869
5000	200	10	0.9344	0.5213
5000	200	20	0.9307	0.3866
5000	200	50	0.9576	0.0951
5000	200	100	0.9646	−0.0014
5000	200	200	0.9606	0.0707
5000	500	10	0.9281	0.4008
5000	500	20	0.9473	0.1617
5000	500	50	0.9567	−0.0151
5000	500	100	0.9576	−0.0256
5000	500	200	0.9575	−0.0010
5000	500	500	0.9579	0.0479
10000	10	10	0.7221	0.9159
10000	20	10	0.8862	0.5513
10000	20	20	0.9746	0.5768
10000	50	10	0.9400	0.5575
10000	50	20	0.9371	0.4408
10000	50	50	0.9467	0.3037
10000	100	10	0.9343	0.5694
10000	100	20	0.9409	0.4156
10000	100	50	0.9467	0.2716
10000	100	100	0.9430	0.1703
10000	200	10	0.9412	0.5188
10000	200	20	0.9386	0.3389
10000	200	50	0.9401	0.2230
10000	200	100	0.9626	0.0505
10000	200	200	0.9640	−0.0035
10000	500	10	0.9340	0.4839
10000	500	20	0.9319	0.3029
10000	500	50	0.9616	−0.0237
10000	500	100	0.9618	−0.0083
10000	500	200	0.9635	−0.0065
10000	500	500	0.9601	−0.0222

Table 3: Results of the simulated experiments (part 2)

# Inputs	Maximum # inputs to same output	Maximum # outputs from same input	Pearson Correlation with Non – Det. Squeeziness	Pearson Correlation with Det. Squeeziness
20000	10	10	0.7207	0.9233
20000	20	10	0.7284	0.5057
20000	20	20	0.8795	0.6133
20000	50	10	0.9577	0.5275
20000	50	20	0.9473	0.4439
20000	50	50	0.9505	0.2777
20000	100	10	0.9359	0.5584
20000	100	20	0.9447	0.3994
20000	100	50	0.9489	0.2609
20000	100	100	0.9546	0.1898
20000	200	10	0.9380	0.5294
20000	200	20	0.9468	0.4245
20000	200	50	0.9527	0.2043
20000	200	100	0.9508	0.1595
20000	200	200	0.9646	0.0686
20000	500	10	0.9380	0.4867
20000	500	20	0.9396	0.3748
20000	500	50	0.9444	0.1971
20000	500	100	0.9659	−0.0063
20000	500	200	0.9622	−0.0113
20000	500	500	0.9645	0.0380
50000	10	10	0.7217	0.9377
50000	20	10	0.5987	0.5300
50000	20	20	0.6437	0.6470
50000	50	10	0.9773	0.3849
50000	50	20	0.9657	0.3799
50000	50	50	0.9574	0.3213
50000	100	10	0.9440	0.5397
50000	100	20	0.9446	0.4274
50000	100	50	0.9558	0.3166
50000	100	100	0.9577	0.1635
50000	200	10	0.9410	0.5575
50000	200	20	0.9442	0.4594
50000	200	50	0.9553	0.2782
50000	200	100	0.9548	0.1574
50000	200	200	0.9489	0.1286
50000	500	10	0.9460	0.5396
50000	500	20	0.9444	0.3939
50000	500	50	0.9505	0.2127
50000	500	100	0.9447	0.1777
50000	500	200	0.9641	0.0243
50000	500	500	0.9657	−0.0021
100000	10	10	0.7612	0.9476
100000	20	10	0.5163	0.5494
100000	20	20	0.3758	0.6118
100000	50	10	0.4458	−0.3832
100000	50	20	0.5332	−0.3582
100000	50	50	0.8772	−0.1103
100000	100	10	0.9846	0.3869
100000	100	20	0.9825	0.3488
100000	100	50	0.9654	0.2656
100000	100	100	0.9648	0.2076
100000	200	10	0.8949	0.7576
100000	200	20	0.9355	0.5278
100000	200	50	0.9547	0.3241
100000	200	100	0.9599	0.1930
100000	200	200	0.9639	0.1122
100000	500	10	0.8638	0.7754
100000	500	20	0.9254	0.5462
100000	500	50	0.9530	0.3266
100000	500	100	0.9536	0.2009
100000	500	200	0.9497	0.0933
100000	500	500	0.9683	0.0262
100000	500	500	0.9651	−0.0247

Table 4: Results of the simulated experiments (part 3)

benchmark[1], that has been widely used to evaluate very heterogeneous algorithms dealing with `FSM`s. Unfortunately, these `FSM`s are deterministic. In order to produce non-deterministic `FSM`s, we follow a methodology used in previous work [20, 24]. Inputs and outputs are strings over three symbols: $\{0, 1, -\}$. If we replace each occurrence of $-$ by 0 and 1, we obtain two different transitions; if the input is the same, then we obtain a non-deterministic `FSM`. For example, from a transition $(q, 10, 0-, q')$ we will obtain two transitions, $(q, 10, 00, q')$ and $(q, 10, 01, q')$, and the resulting `FSM` will be non-deterministic. In Table 5 we display the different `FSM`s with their corresponding number of states, number of transitions before they were extended, and number of transitions after they were extended by replacing the different occurrences of $-$ by 0 and 1.

For these experiments, we used the AutomataLib [25] library to deal with `FSM`s loading, representation, exploration and manipulation. We performed the following steps. Given a length k, our tool explored each `FSM` M of the set and stored all the input and output sequences of length k, as well as the information of which input generates which outputs and which output is generated by which inputs. Then, $\texttt{PColl}_k(M) + \texttt{PDiff}_k(M)$ and $\texttt{NDSq}_k(M)$ values are computed. In the latter case, we use again the Maximum Entropy Principle, thus assuming a uniform probability distribution. Finally, we computed the corresponding Pearson and Spearman correlations. Note that, for each k, this process is completely deterministic: there is no need to repeat it.

We were reducing the number of `FSM`s, out of the 16 initially selected, as we were increasing the value of k. The problem is that due to computational issues, specially memory limitations due to combinatorial explosion, we could not execute all the experiments, for differents k values, in all the experimental subjects. Moreover, `FSM`s with more than 100,000 transitions had to be discarded because the Automatalib library could not load them.

For $k = 2$, we included all the experimental subjects with less than 100,000 transitions. We obtained a Pearson correlation of 0.9303 and a Spearman correlation of 0.9727. These are really good results, given the limitation in the size of the input and output sequences. The next experiment was performed by using all the `FSM`s with less than 1,000 transitions, but expanding the input and output sequences to reach length 7. In this case, we obtained a Pearson correlation of 0.9889 and a Spearman correlation of 0.8571. After this experiment, we performed another one where the experimental subjects were those `FSM`s with less than 100 transitions and allowing input and output sequences of length 9, obtaining a Pearson correlation of 0.5264 and a Spearman correlation of 0.6. Finally, we performed the last experiment, using `FSM`s with less than 50 transitions, computing the input and output sequences up to length 11, and we obtained a Pearson correlation of

[1] `https://people.engr.ncsu.edu/brglez/CBL/benchmarks/index`

Property	bbsse	cse	ex2	ex3	ex5	ex7	keyb	kirkman	lion	mark1	planet	sand	sse	styr	train4	train11
States	16	16	19	10	9	10	19	16	4	15	48	32	16	30	4	11
Det Transitions	56	91	72	36	32	36	170	370	11	22	115	184	56	166	14	25
$Non-Det$ *Transitions*	5264	6528	249	126	86	105	10266	228864	16	254656	321648	323712	5264	398256	17	31

Table 5: FSMs used in the second set of experiments and some of their properties

0.9890 and a Spearman correlation of 0.5. The results of these experiments can be found in Table 6.

With these results we can conclude that our formulation of Non-Deterministic Squeeziness performs really well over FSMs. Nevertheless, it is also affected by the reduced correlation effect. We are aware that this effect is observed only in one case and it is due to one single FSM. A deeper analysis allowed us to conclude that this effect appears because the *amount* of non-determinism is small when compared to the number of inputs. A further discussion about this effect can be found in Section 5.

In these experiments we also computed the mean computation times needed for both Squeeziness and Non-Deterministic Squeeziness. The results are displayed in Table 6, where we show the computation time for Non-Deterministic Squeeziness, for Squeeziness, and for exploring the FSM. As expected, most of the time is used to explore the FSM, while very little time is used to compute the different values of Squeeziness and Non-Deterministic Squeeziness. For example, on average, we need 50.8267 seconds to explore each of the 11 FSMs considered in the experiment where $k = 2$. The total time needed to explore each FSM and compute Non-Deterministic Squeeziness is, on average, equal to 50.8512, that is, less than 0.05% of the total time is used to compute our measure. Moreover, the time needed to compute each of the measures is almost the same. In order to properly compare execution times, we performed a statistical significance test to check whether the execution time of both Squeeziness and Non-Deterministic Squeeziness are *similar*. Specifically, we considered the following null hypothesis: execution times corresponding to both notions follow the same probability distribution. To that end, we performed a one-way ANOVA test[2], that obtained a p-value of 0.97, what represents that there is a 97% probability that the null hypothesis is fulfilled. Therefore, we can accept the null hypothesis with a confidence higher than 95% (its p-value is greater than 0.95). In order to double-check our results, we also performed a t-test and obtained the same p-value. Thus, the conclusion is that the execution times of Non-Deterministic Squeeziness and Squeeziness are similar. These results show that the extra time needed to compute Non-Deterministic Squeeziness is negligible when compared to the time needed to compute Squeeziness. Finally, if we take into account that Non-Deterministic Squeeziness is a conservative extension of Squeeziness, then we can consider using Non-Deterministic Squeeziness even in the deterministic case.

4.4. Research questions answers

We now summarise what the results tell us about the research questions.

Research Question 1. *Is there a strong correlation between Failed Error Propagation and Non-Deterministic Squeeziness? Is this correlation stable along different situations?*

Our results show that the answer to this question is positive: correlation values are always positive and range between 0.3757 and 0.9846. Moreover, we observed that the correlation is stable along different scenarios, with only special cases when the correlation drops due to the great difference between the number of inputs and the amount of non-determinism.

Research Question 2. *Is the correlation between Failed Error Propagation and Non-Deterministic Squeeziness higher than the correlation between Failed Error Propagation and Squeeziness?*

The answer to this question is clearly positive because the correlation of Squeeziness with FEP in a non-deterministic scenario is clearly poor, having even negative correlations for many cases. Therefore, the *update* of Squeeziness to a non-deterministic version was totally necessary.

Research Question 3. *Is there a correlation between Failed Error Propagation and Non-Deterministic Squeeziness when applied over FSMs?*

The answer to this question is also positive, as there is a strong correlation between Non-Deterministic Squeeziness and the addition of PColl and PDiff. The correlations ranged between 0.5264 and 0.9897, improving the results obtained by the simulation experiments.

Research Question 4. *Is the extra computation time needed for Non-Deterministic Squeeziness an important performance issue?*

The answer to this question is negative: the extra computation time is not an important performance issue. Moreover, it is almost negligible when compared to the time needed to explore the FSM. Taking into account the little extra time needed, we can even consider to use Non-Deterministic Squeeziness in the deterministic case, as it will perform

[2]Note that we could use the ANOVA test because we performed an homogeneity of variance check and it raised a positive result.

Length of sequences	#FSMs	Pearson with Non − Det. Squeeziness	Spearman with Non − Det. Squeeziness	Total Time Non − Det. Squeeziness	Total Time Det. Squeeziness	FSM Exploration Time
2	11	0.9303	0.9727	50.8512	50.8272	50.8267
7	7	0.9889	0.8571	34.7928	34.7856	34.7708
9	4	0.5264	0.6000	43.5391	43.4797	43.4213
11	3	0.9890	0.5000	56.0021	54.9940	54.9923

Table 6: Results of the FSM experiments on average (times in seconds)

as well as Squeeziness, with a negligible extra computation time. This is a first step towards a generic tool that can compute the likelihood of having cases of FEP for any FSM, indistinctly of it being deterministic or not.

5. Discussion

There are some points that require further discussion. Specifically, it is important to discuss the decision underlying the definition of Squeeziness. Moreover, although most of the experiments showed good correlations, it is important to investigate the few cases where correlations were not that good. Finally, we would like to address the adequacy of PColl and PDiff to measure FEP and why use them.

5.1. Squeeziness definition

As explained in Section 3, the definition of Non-Deterministic Squeeziness comes from modifying two formulas based on mathematical principles and derivations. Specifically, we remove from these formulas a *correction factor*. Therefore, we cannot claim that Non-Deterministic Squeeziness is the addition of the information loss and gain produced by the FSM. However, our decision to define Non-Deterministic Squeeziness avoiding these correction factors is based on a simple reason: the use of the original formulas (i.e. those including the correction factors) leads to a total sum of 0. Regarding alternatives, we could simply use the original formula of Squeeziness including the correction factor (remember that this factor is equal to 0 for deterministic systems). Unfortunately, preliminary results showed that this is a worse option, concerning correlation with the likelihood of FEP, than our formulation. This behaviour is probably produced by the fact that Squeeziness balances the likelihood of having cases of FEP with the gain of information produced by non-determinism. However, as previously discussed, we should consider that information gain is a new source of FEP. In contrast, our formulation computes the loss of information produced by the collisions and the gain of information produced by the non-determinism and we are adding their effects over FEP, instead of compensating the effect of one with the other.

Finally, one could ask why we are not defining Squeeziness by simply swapping the sign of the correction factor. This *trick* would easily solve the aforementioned problem. However, this alternative formulation does not obtain better correlations (in fact, preliminary experiments showed really bad correlations). We consider that this difference is due to the probability distributions choice: in the formulation of Squeeziness we use the probability distribution over the inputs of the FSM, while in the one corresponding to Alternative Squeeziness we use the probability distribution over the outputs of the FSM. This apparently harmless decision is in fact the key to the success of Non-Deterministic Squeeziness. The reason is that when using the Squeeziness formula, the correction factor (the part addressing the information gained) is determined by the probability of the inputs instead of being determined by the one of the outputs (as in the Alternative Squeeziness formula). The probability induced over the outputs is not necessarily uniformly distributed (because a uniform probability over the inputs does not always generate a uniform probability over the outputs). Thus, the obtained values after computing the formula are different, leading to better empirical results for our Non-Deterministic Squeeziness formulation.

5.2. Correlation Results

It is clear that, for a few cases, we obtain lower correlations than desired. A careful study of the results from the simulations showed that low correlations correspond to systems with low non-determinism and high number of inputs. Our definition of non-deterministic Squeeziness is conservative with respect to the deterministic version of Squeeziness, that is, deterministic FSMs will return the same value with both versions. However, FSMs with really small amounts of non-determinism can produce, due to the logarithmic nature of the formula, a greater influence of the non-deterministic factor than the one supposed to be. We think that this is the main cause of the observed results, but further research is necessary to properly assess this assumption.

5.3. PColl and PDiff

PColl and PDiff are really interesting measures to assess the likelihood of FEP because they are able to estimate the effect of different FEP sources (what we have call collisions and diffusion). A fundamental question is why we are using a more complex formula if we can use these two simpler ones. To answer this question, we present two arguments. First, PColl and PDiff assume uniform distributions but those are not always the best distribution to assess the likelihood of FEP, as explained in Section 3. Specifically, PColl and PDiff are not trivially adaptable to other probability distributions, and thus we would need to develop a different formula for each probability distribution. In contrast, Non-Deterministic Squeeziness is ready to be used with different probability distributions, as they are part of its parameters. Second, there is potential to simplify the Non-Deterministic Squeeziness formula using novel research in Information Theory. Possibilities for adaptation to an approximate computation of Squeeziness come also from the quantifying information flow field, such as a bounded model checking approach [26], a statistical approach to estimate flow quantity [27], and a syntax based approximation [28].

6. Threats to Validity

In order to ensure the validity of the obtained results, we need to address the potential threats that can invalidate them. We start with the threats to internal validity, which refer to uncontrolled factors that can affect the output of the experiments, either in favour or against our hypothesis. The main threat in this category is the possibility of having faults in the code of the experiments. To diminish this threat we carefully tested the code, even using small examples for which we know what were the expected results. Additionally, in order to reduce the impact of the randomness associated with our simulated experiments, we repeated our experiments several times and computed mean values.

The next category, threats to external validity, refers to the generality of our findings in other situations. The main threat in this category is the choice of experimental subjects. As the population of FSMs is unknown, this threat is not fully addressable. However, we used a carefully constructed benchmark that aims to represent real systems to try to diminish this threat.

Finally, the last category is threats to construct validity, which refer to the relevance of the properties we are measuring for the extrapolation of the results to real-world examples. The main threat in this category is how to properly measure the likelihood of having FEP. We addressed this threat using PColl and PDiff for estimating how much FEP is generated due to different processes. An extended discussion about the suitability of these formulas for this task was presented in Section 5. The other big threat in this category is whether the FSMs used in the experiments correspond to possible system components. In order to reduce the impact of this threat, we restricted our range of FSM samples to non-deterministic FSMs obtained from a widely known benchmark, used in many previous research papers.

7. Conclusions and Future Work

Failed Error Propagation (FEP) is an important problem in the Software Testing field: it can hamper the whole testing process by masking faults. Squeeziness has been proposed to assess the likelihood of having FEP in deterministic systems, obtaining a high correlation between these two values. we have extended Squeeziness to deal with non-deterministic FSMs, introducing Non-Deterministic Squeeziness. This proposal combines the potential maximum loss of information and the potential maximum gain of information that a system can produce to assess its likelihood of having FEP.

In order to validate the usefulness of our proposal, we conducted several experiments. We classified those experiments in two categories: simulated experiments and FSM experiments. In both cases, our proposal obtained high correlations with the likelihood of FEP. Moreover, those correlations were stable along different situations. There were a small number of cases where we obtained not so high correlations. After a careful analysis, we concluded that they were produced due to the FSM having a high number of input sequences and a lower amount of potential non-determinism. Thus, we concluded that our proposal is a good tool for assessing FEP.

We also evaluated how our proposal compares with Squeeziness and we found that it outperforms it in non-deterministic systems with only a negligible increment in computation time. Moreover, if we add the fact that our proposal is conservative with respect to the deterministic formula (that is, for a deterministic system it computes the deterministic Squeeziness value), we can conclude that our proposal is a generalisation of the previous notion that can be applied in any system.

For future work, we devise some research lines. The first one focuses on refining our formula so that we solve the problem of the not so high correlations in limit cases. A second line would consider the implementation of this formula in a tool that can automatically assess FEP, extending our previous work [15]. A third line would explore the development of a fully probabilistic version of our framework where we do not assume, by default, uniform distributions. In this case, we need to start with a specification of the SUT that indicates the expected probabilities, quantifying choices between different alternatives, governing the SUT. Since in testing is important to distinguish between inputs and outputs, we will build on top of previous work where we used probabilistic extensions of FSMs [29] and Input Output Transition Systems [30, 31]. Finally, a fourth line would focus on adapting our framework to deal with the non-determinism produced in systems with asynchronous communications [32, 33]. In this line, we would like to evaluate our approach in real IoT architectures where asynchrony plays an important role [34, 35]

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Appendix A. Derivation of Squeeziness formula

Next we present the derivation of the formula presented in Equation 2. First, we consider conditional entropy [23], which tell us that $\mathcal{H}(\xi_{\mathtt{dom}_{M,k}}|\xi_{\mathtt{image}_{M,k}})$ is equal to

$$\sum_{\beta\in\mathtt{image}_{M,k}} \sigma_{\xi_{\mathtt{image}_{M,k}}}(\beta)\cdot\mathcal{H}(\xi_{\mathtt{dom}_{M,k}}|\xi_{\mathtt{image}_{M,k}}=\beta)$$

Next, we apply the notion of conditional probability and take into account that $\xi_{\mathtt{dom}_{M,k}}$ restricted to $\xi_{\mathtt{image}_{M,k}}=\beta$ is the random variable $\xi_{f_M^{-1}(\beta)}$ ranging over $f_M^{-1}(\beta)$ and whose probabilities are equal to

$$\frac{\sigma_{\xi_{\mathtt{dom}_{M,k}}}(\alpha)}{\sigma_{\xi_{\mathtt{dom}_{M,k}}}(f_M^{-1}(\beta))}$$

Therefore, we have that

$$\begin{aligned}
&\mathcal{H}(\xi_{\mathtt{dom}_{M,k}} | \xi_{\mathtt{image}_{M,k}} = \beta) = \\
&\quad = \mathcal{H}(\xi_{f_M^{-1}(\beta)}) \\
&\quad = -\sum_{\alpha \in f_M^{-1}(\beta)} \sigma_{\xi_{f_M^{-1}(\beta)}}(\alpha) \cdot \log_2(\sigma_{\xi_{f_M^{-1}(\beta)}}(\alpha)) \\
&\quad = -\sum_{\alpha \in f_M^{-1}(\beta)} \frac{\sigma_{\xi_{\mathtt{dom}_{M,k}}}(\alpha)}{\sigma_{\xi_{\mathtt{dom}_{M,k}}}(f_M^{-1}(\beta))} \cdot \log_2\left(\frac{\sigma_{\xi_{\mathtt{dom}_{M,k}}}(\alpha)}{\sigma_{\xi_{\mathtt{dom}_{M,k}}}(f_M^{-1}(\beta))}\right)
\end{aligned}$$

Next, we have that $\mathcal{H}(\xi_{\mathtt{dom}_{M,k}} | \xi_{\mathtt{image}_{M,k}})$ is equal to

$$-\sum_{\beta \in \mathtt{image}_{M,k}} \sigma_{\xi_{\mathtt{image}_{M,k}}}(\beta) \cdot \left(\sum_{\alpha \in f_M^{-1}(\beta)} \theta(\alpha) \cdot \log_2(\theta(\alpha))\right) \tag{A.1}$$

where $\theta(\alpha) = \sigma_{\xi_{f_M^{-1}(\beta)}}(\alpha)$. Using an equivalent derivation, we can conclude that the term $\mathcal{H}(\xi_{\mathtt{image}_{M,k}} | \xi_{\mathtt{dom}_{M,k}})$ is equal to

$$-\sum_{\alpha \in \mathtt{dom}_{M,k}} \sigma_{\xi_{\mathtt{dom}_{M,k}}}(\alpha) \cdot \left(\sum_{\beta \in f_M(\alpha)} \theta(\beta) \cdot \log_2(\theta(\beta))\right) \tag{A.2}$$

where $\theta(\beta) = \sigma_{\xi_{f_M(\alpha)}}(\beta)$. Here, the random variable $\xi_{f_M(\alpha)}$ ranges over $f_M(\alpha)$ and its probabilities are equal to

$$\frac{\sigma_{\xi_{\mathtt{image}_{M,k}}}(\beta)}{\sigma_{\xi_{\mathtt{image}_{M,k}}}(f_M(\alpha))} = \frac{\sigma_{\xi_{\mathtt{image}_{M,k}}}(\beta)}{\sum_{\beta \in f_M(\alpha)} \sigma_{\xi_{\mathtt{image}_{M,k}}}(\beta)}$$

In the next step, we apply the *Chain rule* to Equation A.1 and obtain the following expression:

$$\mathcal{H}(\xi_{\mathtt{image}_{M,k}}, \xi_{\mathtt{dom}_{M,k}}) = \mathcal{H}(\xi_{\mathtt{image}_{M,k}}) + \mathcal{H}(\xi_{\mathtt{dom}_{M,k}} | \xi_{\mathtt{image}_{M,k}})$$

where $\mathcal{H}(\xi_{\mathtt{image}_{M,k}}, \xi_{\mathtt{dom}_{M,k}})$ is the joint probability of the two random variables. After another application of the *Chain rule*, in this case to Equation A.2, we obtain the following:

$$\mathcal{H}(\xi_{\mathtt{image}_{M,k}}, \xi_{\mathtt{dom}_{M,k}}) = \mathcal{H}(\xi_{\mathtt{dom}_{M,k}}) + \mathcal{H}(\xi_{\mathtt{image}_{M,k}} | \xi_{\mathtt{dom}_{M,k}})$$

If we combine the previous equalities we obtain

$$\begin{gathered}
\mathcal{H}(\xi_{\mathtt{image}_{M,k}}) + \mathcal{H}(\xi_{\mathtt{dom}_{M,k}} | \xi_{\mathtt{image}_{M,k}}) \\
\| \\
\mathcal{H}(\xi_{\mathtt{dom}_{M,k}}) + \mathcal{H}(\xi_{\mathtt{image}_{M,k}} | \xi_{\mathtt{dom}_{M,k}})
\end{gathered}$$

This is the formula that we showed in Equation 1. Now we can rewrite Squeeziness as

$$\mathtt{Sq}_k(M) = \mathcal{H}(\xi_{\mathtt{dom}_{M,k}} | \xi_{\mathtt{image}_{M,k}}) - \mathcal{H}(\xi_{\mathtt{image}_{M,k}} | \xi_{\mathtt{dom}_{M,k}})$$

Using Equations A.1 and A.2 and the fact that

$$\sigma_{\xi_{\mathtt{image}_{M,k}}}(\beta) = \sum_{\alpha \in f_M^{-1}(\beta)} \sigma_{\xi_{\mathtt{dom}_{M,k}}}(\alpha)$$

we can finally rewrite Squeeziness as

$$\begin{aligned}
\mathtt{Sq}_k(M) = \; & -\sum_{\beta \in \mathtt{image}_{M,k}} \left(\sum_{\alpha \in f_M^{-1}(\beta)} \sigma_{\xi_{\mathtt{dom}_{M,k}}}(\alpha)\right) \cdot \mathcal{R}_M(\beta) \\
& + \sum_{\alpha \in \mathtt{dom}_{M,k}} \sigma_{\xi_{\mathtt{dom}_{M,k}}}(\alpha) \cdot \mathcal{R}''_M(\alpha)
\end{aligned}$$

where the term $\mathcal{R}_M(\beta)$ is equal to

$$\sum_{\alpha \in f_M^{-1}(\beta)} \frac{\sigma_{\xi_{\mathtt{dom}_{M,k}}}(\alpha)}{\sigma_{\xi_{\mathtt{dom}_{M,k}}}(f_M^{-1}(\beta))} \cdot \log_2\left(\frac{\sigma_{\xi_{\mathtt{dom}_{M,k}}}(\alpha)}{\sigma_{\xi_{\mathtt{dom}_{M,k}}}(f_M^{-1}(\beta))}\right)$$

and the term $\mathcal{R}''_M(\alpha)$ is equal to

$$\sum_{\beta \in f_M(\alpha)} \frac{\sum_{\alpha \in f_M^{-1}(\beta)} \sigma_{\xi_{\mathtt{dom}_{M,k}}}(\alpha) \cdot \log_2\left(\frac{\sum_{\alpha \in f_M^{-1}(\beta)} \sigma_{\xi_{\mathtt{dom}_{M,k}}}(\alpha)}{\sum_{\beta \in f_M(\alpha)} \sum_{\alpha \in f_M^{-1}(\beta)} \sigma_{\xi_{\mathtt{dom}_{M,k}}}(\alpha)}\right)}{\sum_{\beta \in f_M(\alpha)} \sum_{\alpha \in f_M^{-1}(\beta)} \sigma_{\xi_{\mathtt{dom}_{M,k}}}(\alpha)}$$

Appendix B. Proof of Corollary 1

In this appendix we give the proof of Corollary 1. This result can ge used to deduce an alternative characterisation of Non-Deterministic Squeeziness.

Corollary 1. Let $M = (Q, q_{in}, I, O, T)$ be an FSM and $k > 0$. Let us consider a random variable $\xi_{\mathtt{image}_{M,k}}$ ranging over the image of $f_{M,k}$. We have that

$$\mathtt{AlSq}_k(M) = -\sum_{\alpha \in \mathtt{dom}_{M,k}} \left(\sum_{\beta \in f_M(\alpha)} \sigma_{\xi_{\mathtt{image}_{M,k}}}(\beta)\right) \cdot \mathcal{R}'_M(\alpha) + \mathcal{S}'_M$$

where the term $\mathcal{R}'_M(\alpha)$ is equal to

$$\sum_{\beta \in f_M(\alpha)} \frac{\sigma_{\xi_{\mathtt{image}_{M,k}}}(\beta)}{\sigma_{\xi_{\mathtt{image}_{M,k}}}(f_M(\alpha))} \cdot \log_2\left(\frac{\sigma_{\xi_{\mathtt{image}_{M,k}}}(\beta)}{\sigma_{\xi_{\mathtt{image}_{M,k}}}(f_M(\alpha))}\right)$$

and the term $\mathcal{S}'_M$ is given in Figure 3.

Proof. We start with the definition of conditional entropy [23], which tell us that $\mathcal{H}(\xi_{\mathtt{image}_{M,k}} | \xi_{\mathtt{dom}_{M,k}})$ is equal to

$$\sum_{\alpha \in \mathtt{dom}_{M,k}} \sigma_{\xi_{\mathtt{dom}_{M,k}}}(\alpha) \cdot \mathcal{H}(\xi_{\mathtt{image}_{M,k}} | \xi_{\mathtt{dom}_{M,k}} = \alpha)$$

Next, we apply the notion of conditional probability and take into account that $\xi_{\mathtt{image}_{M,k}}$ restricted to $\xi_{\mathtt{dom}_{M,k}} = \alpha$ is the random variable $\xi_{f_M(\alpha)}$ ranging over $f_M(\alpha)$ and whose probabilities are equal to

$$\frac{\sigma_{\xi_{\mathtt{image}_{M,k}}}(\beta)}{\sigma_{\xi_{\mathtt{image}_{M,k}}}(f_M(\alpha))}$$

Therefore, we have that

$$\begin{aligned}
&\mathcal{H}(\xi_{\texttt{image}_{M,k}} | \xi_{\texttt{dom}_{M,k}} = \alpha) = \\
&\quad = \mathcal{H}(\xi_{f_M(\alpha)}) \\
&\quad = -\sum_{\beta \in f_M(\alpha)} \sigma_{\xi_{f_M(\alpha)}}(\beta) \cdot \log_2(\sigma_{\xi_{f_M(\alpha)}}(\beta)) \\
&\quad = -\sum_{\beta \in f_M(\alpha)} \frac{\sigma_{\xi_{\texttt{image}_{M,k}}}(\beta)}{\sigma_{\xi_{\texttt{image}_{M,k}}}(f_M(\alpha))} \cdot \log_2\left(\frac{\sigma_{\xi_{\texttt{image}_{M,k}}}(\beta)}{\sigma_{\xi_{\texttt{image}_{M,k}}}(f_M(\alpha))}\right)
\end{aligned}$$

An immediate consequence is that $\mathcal{H}(\xi_{\texttt{image}_{M,k}} | \xi_{\texttt{dom}_{M,k}})$ is equal to

$$-\sum_{\alpha \in \texttt{dom}_{M,k}} \sigma_{\xi_{\texttt{dom}_{M,k}}}(\alpha) \cdot \left(\sum_{\beta \in f_M(\alpha)} \theta(\beta) \cdot \log_2(\theta(\beta)) \right) \tag{B.1}$$

where $\theta(\beta) = \sigma_{\xi_{f_M(\alpha)}}(\beta)$. Using an equivalent derivation, we can conclude that the term $\mathcal{H}(\xi_{\texttt{dom}_{M,k}} | \xi_{\texttt{image}_{M,k}})$ is equal to

$$-\sum_{\beta \in \texttt{image}_{M,k}} \sigma_{\xi_{\texttt{image}_{M,k}}}(\beta) \cdot \left(\sum_{\alpha \in f_M^{-1}(\beta)} \theta(\alpha) \cdot \log_2(\theta(\alpha)) \right) \tag{B.2}$$

where $\theta(\alpha) = \sigma_{\xi_{f_M^{-1}(\beta)}}(\alpha)$. Here, the random variable $\xi_{f_M^{-1}(\beta)}$ ranges over $f_M^{-1}(\beta)$ and its probabilities are equal to

$$\frac{\sigma_{\xi_{\texttt{dom}_{M,k}}}(\alpha)}{\sigma_{\xi_{\texttt{dom}_{M,k}}}(f_M^{-1}(\beta))} = \frac{\sigma_{\xi_{\texttt{dom}_{M,k}}}(\alpha)}{\sum_{\alpha \in f_M^{-1}(\beta)} \sigma_{\xi_{\texttt{dom}_{M,k}}}(\alpha)}$$

Now, if we apply the *Chain rule* to Equation B.1 then we have

$$\mathcal{H}(\xi_{\texttt{dom}_{M,k}}, \xi_{\texttt{image}_{M,k}}) = \mathcal{H}(\xi_{\texttt{dom}_{M,k}}) + \mathcal{H}(\xi_{\texttt{image}_{M,k}} | \xi_{\texttt{dom}_{M,k}})$$

where $\mathcal{H}(\xi_{\texttt{dom}_{M,k}}, \xi_{\texttt{image}_{M,k}})$ is the joint probability of the two random variables. Applying the *Chain rule* to Equation B.2, we also have

$$\mathcal{H}(\xi_{\texttt{dom}_{M,k}}, \xi_{\texttt{image}_{M,k}}) = \mathcal{H}(\xi_{\texttt{image}_{M,k}}) + \mathcal{H}(\xi_{\texttt{dom}_{M,k}} | \xi_{\texttt{image}_{M,k}})$$

Combining the previous equalities we obtain

$$\begin{gathered}
\mathcal{H}(\xi_{\texttt{dom}_{M,k}}) + \mathcal{H}(\xi_{\texttt{image}_{M,k}} | \xi_{\texttt{dom}_{M,k}}) \\
\| \\
\mathcal{H}(\xi_{\texttt{image}_{M,k}}) + \mathcal{H}(\xi_{\texttt{dom}_{M,k}} | \xi_{\texttt{image}_{M,k}})
\end{gathered}$$

and this is the formula given in Equation 1. Now we can rewrite Alternative Squeeziness as

$$\texttt{AlSq}_k(M) = \mathcal{H}(\xi_{\texttt{image}_{M,k}} | \xi_{\texttt{dom}_{M,k}}) - \mathcal{H}(\xi_{\texttt{dom}_{M,k}} | \xi_{\texttt{image}_{M,k}})$$

Using Equations B.1 and B.2 and the fact that

$$\sigma_{\xi_{\texttt{dom}_{M,k}}}(\alpha) = \sum_{\beta \in f_M(\alpha)} \sigma_{\xi_{\texttt{image}_{M,k}}}(\beta)$$

we can finally rewrite Alternative Squeeziness as

$$\begin{aligned}
\texttt{AlSq}_k(M) = \; & -\sum_{\alpha \in \texttt{dom}_{M,k}} \left(\sum_{\beta \in f_M(\alpha)} \sigma_{\xi_{\texttt{image}_{M,k}}}(\beta) \right) \cdot \mathcal{R}'_M(\beta) \\
& + \sum_{\beta \in \texttt{image}_{M,k}} \sigma_{\xi_{\texttt{image}_{M,k}}}(\beta) \cdot \mathcal{R}'''_M(\alpha)
\end{aligned}$$

where the term $\mathcal{R}'_M(\beta)$ is equal to

$$\sum_{\beta \in f_M(\alpha)} \frac{\sigma_{\xi_{\texttt{image}_{M,k}}}(\beta)}{\sigma_{\xi_{\texttt{image}_{M,k}}}(f_M(\alpha))} \cdot \log_2\left(\frac{\sigma_{\xi_{\texttt{image}_{M,k}}}(\beta)}{\sigma_{\xi_{\texttt{image}_{M,k}}}(f_M(\alpha))}\right)$$

and the term $\mathcal{R}'''_M(\alpha)$ is given by the following expression:

$$\sum_{\alpha \in f_M^{-1}(\beta)} \frac{\sum_{\beta \in f_M(\alpha)} \sigma_{\xi_{\texttt{image}_{M,k}}}(\beta) \cdot \log_2\left(\frac{\sum_{\beta \in f_M(\alpha)} \sigma_{\xi_{\texttt{image}_{M,k}}}(\beta)}{\sum_{\alpha \in f_M^{-1}(\beta)} \sum_{\beta \in f_M(\alpha)} \sigma_{\xi_{\texttt{image}_{M,k}}}(\beta)} \right)}{\sum_{\alpha \in f_M^{-1}(\beta)} \sum_{\beta \in f_M(\alpha)} \sigma_{\xi_{\texttt{image}_{M,k}}}(\beta)}$$